Commentary

In My Opinion

Compute and Conjecture

When I was a mathematical child, abstraction was king. In algebra (my field, and one I will concentrate on here) I was taught commutative algebra before I knew what a number field was. Galois theory was abstract structure; actual groups and fields did not appear until after the proof of the fundamental theorem.

It was as if examples were the detritus, and proof and theorem the real thing. Computation was for those who could not think abstractly—and we all knew that mathematics was abstraction. (This was most obvious in the differential equations course, where existence proofs formed the basis of the math department's offering. Learning to solve them was relegated to engineering.) Examples served to illustrate results and were not a way of doing mathematics. They did not guide research, or its direction, or lead to conjectures.

Before this century mathematicians computed aplenty. From Pythagoras to Archimedes, the Greeks calculated. Astronomical reckonings drove much of Newton's, Euler's, Gauss's, and Poincaré's work. Euler computed his way to quadratic reciprocity. Gauss's extensive calculations led him to the prime number theorem.[1] Based on computational work, Dedekind and Frobenius conjectured many results concerning group representations. Ramanujan used calculations to guide his many conjectures. But after Hilbert demonstrated the power of abstract methods in the basis and syzygy theorems and the Nullstellensatz, computation fell out of favor. Noether's work furthered the ascendancy of abstract methods. Abstraction replaced computation, and mathematics grew richer—but poorer too.

Abstract understanding is like viewing terrain through a satellite map, while examples show what the land is really like under your feet. Research benefits from both approaches. Knowing the terra firma often demonstrates why and where the theorem is true. Computation can help uncover surprising connections, and it can uncover fruitful areas for study.

Knowledge of the individual examples has always been crucial to group theory. It led, for example, to Burnside's conjecture (now the Feit-Thompson theorem) that every simple nonabelian group is of even order. More recently Tits's theory of buildings was derived from an intimate understanding of the structure of many groups. Birch and Swinnerton-Dyer used their knowledge of many elliptic curves obtained by extensive calculations to hone their famous conjecture. And people found questions to ask about the Mandlebrot set by looking at the pictures—then they went and proved theorems.

If for most of this century computation was held in low esteem, now the pendulum appears to be swinging back. It is doing so at a propitious time. Although mathematics has grown more complex than a century ago, we have more computational machinery than our predecessors did.

Symbolic computation packages such as AXIOM, Derive, GAP, Grobner, Macaulay, MAGMA, Maple, Mathematica, Pari/GP, and Singular[2] have made many calculations far easier to perform. Systems developed by mathematical computer scientists and computational mathematicians enable us to easily factor polynomials, solve systems of polynomial equations, compute Galois groups, build groups out of smaller ones (e.g., compute wreath products), calculate the primary decomposition of an ideal, perform arithmetic in rational function fields, compute algebraic varieties, enumerate the partitions of a set, construct Goppa and Reed-Muller codes, build graphs out of smaller ones, do symbol splitting in symbolic dynamics, compute limits, Taylor series, and Laplace transforms, and exactly solve ODEs and PDEs. One can check whether a polynomial is irreducible over $\mathbb{Q}$, compute its Galois group, find subgroups of the Galois group, then compute the corresponding subfields of the splitting fields. In the finite case, one can construct a group from a set of generators and determine its commutator subgroup and its Sylow subgroups. One can construct a graph, determine its automorphism group, and then construct the composition factors of the automorphism group. All of this can be done easily using various symbolic computation programs. (In a satisfying cross-fertilization, the work in symbolic computation has led to new mathematical results in algebra, analysis, combinatorics, and logic.)

In Hilbert's time multivariate computations grew too quickly to be computed by hand; now many of these problems can be easily done by computer. We can solve harder problems, in extensions of higher degree, with more variables than we could handle a decade ago.

Computation and examples enrich and guide research as much as they do teaching. At a time when mathematicians are returning to computation, computers and symbolic computation programs are giving mathematicians an exciting opportunity to expand their research capabilities.

—Susan Landau
Associate Editor

[1] *Gauss had a very modern view of computational complexity. In Art. 329 of* Disquisitiones, *which is about factorization, he carefully distinguishes primality testing from factorization and says things like "It is in the nature of the problem that* any *method will become more prolix as the numbers get larger. Nevertheless, in the following methods, the difficulties increase rather slowly, and numbers with 7, 8, or even more digits have been handled with success and speed beyond expectation … ."*

[2] *See* `http://symbolicnet.mcs.kent.edu/www-sites.html#A1.1` *for a partial listing of symbolic computation systems.*

Letters to the Editor

Impostures Intellectuelles and Faris's Review

I was very interested in the book review article by William G. Faris of *Impostures Intellectuelles* (Editions Odile Jacob, Paris, 1997) by Alan Sokal and Jean Bricmont in the August 1998 issue of the *Notices* (Vol. 45 pp. 874–876). The reason for my interest is the following point:

"The major gap in the Sokal-Bricmont book is that it avoids dealing with…the confusion over the foundations of quantum mechanics. This confusion is a major weak point in modern physical science. Numerous popular writings about science exploit this obscurity, but the book does not address this issue."

To the best of my knowledge, no other reviewer made this important point, and I suggest that this is a grave omission by both the Sokal-Bricmont book and its numerous reviews.

It is generally believed that postmodernism was originated by culture studiers in the revolutionary ambience of 1960s' France. But from which prior paradigms might the French postmodernists have derived their (now rightly recognised as) daft ideas? Might they have been influenced by the philosophical utterances of earlier eminent mathematicians and scientists (mostly quantum physicists)? I have argued that this is indeed the case. The following passages are conveniently taken from a single source, Alan L. Mackay's *A Dictionary of Scientific Quotations* (Adam Hilger, Bristol, 1991):

Niels Bohr: "…two sorts of truth: trivialities, where opposites are obviously absurd, and profound truths, recognised by the fact that the opposite is also a profound truth."

J. B. S. Haldane: "The universe is not only queerer than we suppose, but queerer than we can suppose."

David Bohm: "There are no things, only processes."

Hermann Bondi: "[Science doesn't deal with facts; indeed] fact is an emotion-loaded word for which there is little place in scientific debate."

Bertrand Russell: "Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true."

G. H. Hardy: "Beauty is the first test; there is no permanent place in the world for ugly mathematics."

Paul Dirac: "It is more important to have beauty in one's equations than to have them fit experiment."

Arthur Eddington: "It is also a good rule not to have overmuch confidence on the observational results that are put forward until they are confirmed by theory."

Albert Einstein: "Imagination is more important than knowledge."

Freeman Dyson: "Most of the papers which are submitted to the *Physical Review* are rejected, not because it is impossible to understand them, but because it is possible. Those which are impossible to understand are usually published."

Fred Hoyle: "[We must] recognise ourselves for what we are—the priests of a not very popular religion."

Before any mathematicians and scientists (and especially quantum physicists) dare to accuse any others of the very serious charge of intellectual imposture, they first ought to put their own houses in order.

—Theo Theocharis
London, England

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More History of the Chowla-Selberg Formula

Although the authors of the Chowla memorial article (*Notices*, May 1998) did not want to pursue the history of the Chowla-Selberg formula, there is more to it than is indicated in the exchange of letters in the September *Notices*. The relevant information can be found in section 3.2 of N. Schappacher's book *Periods of Hecke Characters* (Lecture Notes in Mathematics, vol. 1301, Springer-Verlag).

It seems that historically there were two lines of investigation connected with the formula. One involves examples due to Legendre and Eisenstein, and Chowla and Selberg were aware of this part of the history. But the formula itself was discovered about fifty years before the first article of Chowla and Selberg. Building on work of Berger and Kronecker, M. Lerch proved the Chowla-Selberg formula in an article "Sur quelques formules relatives au nombre des classes" (*Bull. Sci. Math.* (2) **21** (1897), prem. partie, 290–304). In a 1903 paper Landau proved a formula that implies the Lerch result, and then Schappacher is able to find no reference to the Lerch or Landau result in the literature between 1903 and the first paper of Chowla and Selberg in 1949. This historical information came to light when R. Sczech pointed out the Landau paper to Schappacher.

Schappacher goes on to describe how Weil looked, with only partial success, for a geometric proof of the formula, at least up to a rational factor, and then how B. H. Gross found the result mentioned in the September letter by McGuinness. More precisely, the method of Gross recovers the formula only up to a rational factor. This factor was determined by P. Colmez ("Périodes des variétés abéliennes à multiplication complexe", *Ann. of Math.* **138** (1993), 625–683), using the theory of p-adic periods.

—Jan Nekovář
University of Cambridge

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Consider Price When Adopting Textbooks

I would like to comment on Edwin Beschler's November article, "The Pricing of Scientific Publications", and the increasing trend of textbook prices in general. One aspect of commercial publishing which is avoided in Beschler's article is lower-division undergraduate textbook prices. His essay doesn't account for why a student pays $100 for a "best-selling" calculus text. Although these books usually contain more colors and pictures than the lower-selling advanced texts, they can be printed in much higher quantities and shouldn't have the same long-term storage costs. Moreover, since they are popular texts, it seems that the risk is much lower. In short, it appears that the total unit cost, etc., doesn't account for the price, and it seems to follow that publishers are indeed price gouging.

Notice that most of these popular texts are in multiple editions. Although it speaks well of the authors for writing books that stand the test of time, it appears that many of these texts undergo revision every two to three years. It follows that with every new edition, the bookstores' used inventories are wiped out, forcing students to purchase new books. Hence it is in the publisher's best interest to be in the process of constant revision and to urge departments to switch texts. I fear that much of the contention in recent years regarding reformed pedagogy and technology-based learning has been promoted, even fueled by publishers. Authors are perhaps pressured to write specialized variations of their textbooks to accommodate specific calculators, classical and reformed teaching styles, single-semester and tracked courses, etc., thus again hampering and complicating the used-book market and passing a further expense on to the students.

I would like to urge teachers and book search committees in general to be more mindful of the extra expense their students incur when they switch either texts or editions and perhaps to factor into their choice the expected duration that a book, as is, will remain in print. We shouldn't be anti-publisher, but contrary to the tone of Beschler's article, they need us as much as we need them, and we shouldn't think that our book reps are doing us any favors. As teachers we have the opportunity and obligation to respond as consumers even though we aren't the ones consuming.

—Jeffrey Humpherys
Indiana University, Bloomington

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Beschler Replies

As a parent currently putting his third child through college, I cannot help but share Jeffrey Humpherys's consternation at $100 textbooks. In the limited space available to me, I did indeed skirt that issue in favor of an emphasis on short-run, research-level books, which share some dynamics with journals.

My main objection to Humpherys's letter is that it again places all the blame on publishers, as though authors and the educational establishment had nothing to do with the requirements that have been put on textbooks to make them acceptable, as though the publishers create these "products" and the captive market has no choice but to pay the bill for them. There is something fundamentally wrong with this picture. There is no doubt that some publishers some of the time reap excessive profits on some of their products. Others settle for less. I should also mention that, since textbooks at the calculus level are not usually published by university presses, the comparison between profit and nonprofit organizations does not apply here. But the principle of attempting to profit on investment, as any business must, remains intact.

Some of the investment that goes into a textbook includes: extensive reviews to determine that the book has all the qualities of the market leader and surpasses them; supplemental material that accompanies the book gratis (answer manuals, teachers' manuals, tapes, CDs, videos, etc.); competitive royalties to attract the author in the first place; copious numbers of free copies to potential adopters; extensive sales forces to bring the wonders of this new "product" to the ears of adoption committees already deluged by similar presentations; marketing "gimmicks" to further increase visibility; and so on. In addition there are such items as checking to ensure that no words in the text exceed a certain presumed reading level, all concepts included are "politically correct", and other excesses at the extreme end of what is happening in textbook publishing. All of these activities are aimed to penetrate a market that has been shaped by the educational establishment and its perceived needs. They are in answer to what the market continually demonstrates that it wants.

A great deal of this investment must be recouped during the first year of a textbook's expected life, since the used-book marketers have greatly developed their ability to collect used copies and direct them to the most likely re-users. In addition, the same investment must be made for a textbook that fails to capture a significant part of the market and whose costs must therefore be subsumed within the economics of the successful ones. The fact that most books lose money and are subsidized by the few that succeed is well known in all areas of market-driven publishing.

If the educational establishment wishes to change the rules of the game, it will follow as the night the day that publishers will adapt. Pressure on authors and urgent arguments to adopting committees are part of the dynamics, but without willing cooperation of these partners in the enterprise, publishers would be powerless. In the end, I agree with Humpherys that the teachers, albeit not being the consumers themselves, ought to be the primary voice in determining what tools they want in the teaching of our children. How they can best exercise their "responsibility and obligation" in this process ought to be the subject of ongoing debate within the educational establishment and should involve the publishers. To paraphrase a comment from my article, "If commercial publishers are part of the problem, they CAN also be part of the solution."

—Edwin F. Beschler
Boston, MA

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