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Pricing of American retail options

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Abstract— **We continue our exploration in the use of option contracts as a means of managing and controlling inventories in a retail market. We propose a new class of American put option contracts on inventories of retail goods, where the retailer can exercise the option at any time during the contract period, thus requiring that the option writer purchase any unsold inventory at a specified strike price. However, to improve market efficiency this option contract allows the retailer to freely adjust the sale price of the underlying good throughout the contract period. As the retailer is expected to select an optimal pricing policy for the goods, the options can be priced accordingly.**

I. INTRODUCTION

In recent years, the capital markets have evolved considerably in the scope, use, and volume of financial derivatives. In addition to the usual exchange-traded instruments such as stocks, bonds, futures, and options, there are also numerous, and often complex, over-the-counter derivatives and other investment vehicles used to transfer cash flows and risks amongst traders around the world. The recent *Financial Crisis of 2008* and the corresponding *Global Recession of 2009*, caused in part by the overexposure to poorly-understood collateralized debt obligations, mortgage-backed securities, and subprime mortgages, tells us that society clearly still has much to learn about financial engineering and the correct quantification of risk. Nonetheless, despite the ever-growing list of blunders and debacles, businesses in the aggregate seem to enjoy increasingly greater access to investment capital, less exposure to market risk, increased liquidity, and higher productivity as a result of this evolution in derivative securities and investment banking [4], [5].

In the retail markets, merchants hold inventories of goods and services much like investors hold portfolios of investment securities. Although options have been around for centuries in the financial and commodity markets [6], they have only recently been suggested for use in a retail environment [2], [3], [1]. In this paper, we propose a new type of American retail put option contract, where the retailer (option holder) has the right, but not the obligation, to sell all her remaining inventory to the option writer at any time during the contract period at a predetermined price, called the strike price. We further allow the retailer the freedom to adjust the sale price continuously throughout the contract period to control for demand in an attempt to maximize profits. We provide a model for customer purchases as a function of the retailer's pricing policy, and determine the option's value accordingly. In a companion paper [1], we considered the European retail put option, which only allowed the retailer to exercise the contract at the end of the period. The added flexibility in the American case makes the pricing problem significantly more complicated since we have to determine the optimal stopping time for the retailer to exercise the option.

The American retail put option is particularly useful when the retailer is overstocked and would be better off immediately liquidating inventory rather than holding off until the end of the contract period. This might happen in situations where the underlying good experiences a substantial change in demand, where the retailer's profit margins become smaller than the rate of interest, or where the retailer needs to acquire cash immediately. The American retail put option might also be useful in instances where the demand rates drop, decay, or fluctuate cyclically, as in the case of perishable, depreciable, and seasonal goods respectively.

In Section II, we present the main result of this paper, which is an algorithm for determining the risk-neutral valuation of an American retail option where the stochastic demand is a non-homogenous Poisson process with a known arrival rate that depends on the sale price. Although we only consider arrivals that depend linearly or log-linearly on the price, the method presented is quite general. The algorithm depends first on computing the retailer's optimal pricing policy, which in turn gives the Poisson arrival rates. These are used to calculate the expected remaining revenue at each point in time, which, when compared with the value of exercising the option, indicates when early exercise is more profitable than holding on to the option. In the final step, the Poisson arrival rates determine the inventory probabilities at times of early exercise, which we use to price the option; see Section III. We analyze the sensitivity of the option to input parameters in Section IV, then in Section V we consider the benefits of the American put option with variations in demand rates and payoff policies. We conclude this paper in Section VI with a discussion of open problems.

II. PROBLEM FORMULATION

In this section we formulate the American retail option pricing problem. We assume throughout that the demand rate $\{X(t)\}_{t\geq 0}$ is a time dependent non-homogeneous Poisson process, with arrival rate $\lambda(u(t))$, where $u(t)$ is the sale price set by the retailer at time t. We discretize the contract period $[0,T]$ into n equally spaced intervals $[t, t+\Delta t)$, with $\Delta t = T/n$ and n large. If $u(t)$ is fixed on each subinterval, the

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Poisson process is then described by

$$\mathbb{P}(X(t+\Delta t) = p+q | X(t) = p) \tag{1}$$
$$= \begin{cases} 1-\lambda(u(t))\Delta t + o(\Delta t) & \text{if } q=0 \\ \lambda(u(t))\Delta t + o(\Delta t) & \text{if } q=1 \\ o(\Delta t) & \text{if } q>1, \end{cases}$$

where $o(\Delta t)$ is the standard little-o notation.

We use the notation I_i for the inventory level during period i; thus I_0 represents the initial inventory. I_i is a discrete valued stochastic process where $I_i = (I_0 - X(i\Delta t))^+$. We remark that the inventory level is positive and monotonically decreasing since there are no returns or backlogs allowed in our model, nor can the retailer order more inventory. This also implies that the zero state is absorbing i.e., if $I_{i_0} = 0$ for some period i_0 then $I_i = 0$ for all $i > i_0$.

A. Inventory Probabilities

Recall that an American put option contract gives the option holder the right to sell all remaining inventory to the option writer anytime during the contract period. If the option holder exercises the option, then the writer pays the buyer the strike price for each unsold inventory and retrieves it. To prevent arbitrage, the premium paid to the option writer must correspond to the risk-neutral expected value of the put option contract. We will show it can be found using $P_{i,j} = \mathbb{P}(I_i = j)$, the probability that the inventory level is j units during period i. Then $\sum_{j=0}^{I_0} P_{i,j} = 1$ for each i and $0 \le P_{i,j} \le 1$ for all (i,j).

Let $u_{i,j}$ be the sale price during period i for inventory level j. We can solve for $P_{i,j}$ for all (i,j) by fixing $u_{i,j}$ during each period i so that for $j > 0$

$$\mathbb{P}(I_{i+1} = j | I_i = j) = 1 - \lambda(u_{i,j})\Delta t + o(\Delta t) \tag{2a}$$
$$\mathbb{P}(I_{i+1} = j | I_i = j+1) = \lambda(u_{i,j+1})\Delta t + o(\Delta t) \tag{2b}$$
$$\mathbb{P}(I_{i+1} = j | I_i \ge j+2) = o(\Delta t). \tag{2c}$$

The law of total probability states that if $B_0, B_1, \ldots, B_n$ is a partition of a probability space then for any event A, $\mathbb{P}(A) = \sum_{m=0}^{n} \mathbb{P}(A|B_m)\mathbb{P}(B_m)$. Combining these facts, we get $\mathbb{P}(I_{i+1} = j) = \sum_{m=0}^{I_0} \mathbb{P}(I_{i+1} = j | I_i = m)\mathbb{P}(I_i = m)$. As shown in Figure 1, $P_{i+1,j}$ is the average of the probabilities of its contributing states weighted by the probabilities that those states transition to the state $(i+1, j)$. Finally,

$$P_{i+1,j} = \begin{cases} P_{i,0} + \lambda(u_{i,1})\Delta t P_{i,1} + o(\Delta t) & \text{for } j=0 \\ (1-\lambda(u_{i,j})\Delta t)P_{i,j} & \\ \quad +\lambda(u_{i,j+1})\Delta t P_{i,j+1} + o(\Delta t) & \text{for } 0<j<I_0 \\ (1-\lambda(u_{i,I_0})\Delta t)P_{i,I_0} + o(\Delta t) & \text{for } j = I_0, \end{cases} \tag{3}$$

since $P_{i,I_0+1} = 0$.

B. Expected Remaining Revenue

We define D_i to be the (price dependent) demand function during period i, which by (2) becomes

$$D_i = \begin{cases} 0 & \text{w prob. } 1-\lambda(u_{i,j})\Delta t + o(\Delta t) \\ 1 & \text{w prob. } \lambda(u_{i,j})\Delta t + o(\Delta t) \\ \ge 2 & \text{w prob. } o(\Delta t), \end{cases} \tag{4}$$

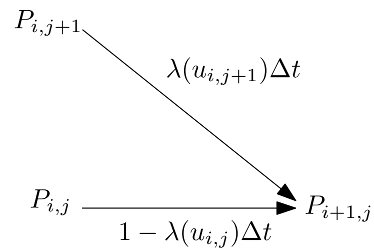


Fig. 1. Single period branch describing (3).

if the current inventory level I_i is j. Since it is possible for the demand to exceed the inventory available for sale, we let S describe the amount of inventory sold during period i. We define $S(I_i, D_i) = \min\{I_i, D_i\}$. We remark that $S(0, D_i) = 0$ for all D_i and $S(I_i, 0) = 0$ for all I_i since no inventory is sold if there is no inventory or no demand.

Because the American put option contract gives the retailer the ability to exercise the option anytime before the expiration date, solving for the value of the option becomes a dynamic programming problem. The retailer must assess at each period i before the option expires whether immediate exercise will garner more profit than is expected by holding on to the option. We must consider the retailer's expected revenue since her actual revenue is not a predictable process. Consequently, we will suppose that the retailer automatically exercises the option at the end of the contract period (at time T) on all remaining inventory. Based on that supposition we will evaluate the retailer's expected remaining revenue, which we will update to the immediate payout if early exercise earns the retailer more profit. We will also presume that the pricing policy $\{u_{i,j}\}$ is given, although we will compute the optimal pricing policy in Section II-E. The total revenue is

$$R = K(I_0 - \sum_{l=0}^{n-1} D_l)^+ + \sum_{k=0}^{n-1} u_{k,I_k} e^{r(T-k\Delta t)} S(I_k, D_k), \tag{5}$$

where r is the rate of interest, which we assume is constant, and K is the unit strike price. For comparison purposes, the unit sale price is converted into its future value at time T. We set the total demand D equal to $\sum_{l=0}^{n-1} D_l$ for simplification. Because we wish to model a dynamic program, it is important that we are able to calculate the remaining revenue at any period i. We define the remaining revenue (value function) from period i onward by

$$R(i) = K(I_0 - D)^+ + \sum_{k=i}^{n-1} u_{k,I_k} e^{r(T-k\Delta t)} S(I_k, D_k). \tag{6}$$

We let $R_{i,j} = R(i)$ when $I_i = j$. It follows that $R_{i,0} = 0$. We set $R_{n,j} = jK$, the payoff of the option at expiration.

Since $\mathbb{P}(D_i = 1) = \lambda(u_{i,j})\Delta t + o(\Delta t)$ is small and n is large, we can simplify our calculation of the expected remaining revenue by using the Poisson approximation to the binomial distribution. Note that $\Delta t = T/n \xrightarrow{n\to\infty} 0$. Thus the $o(\Delta t)$ function is close to zero for large n so we can ignore it. Using the law of total expectation we write

$$\begin{aligned}\mathbb{E}[R_{i,j}] &= \mathbb{E}[\mathbb{E}[R_{i,j}|D_i]] \qquad (7)\\ &= \mathbb{E}[R_{i,j}|D_i=1]\mathbb{P}(D_i=1) + \mathbb{E}[R_{i,j}|D_i=0]\mathbb{P}(D_i=0).\end{aligned}$$

Then when $j > 0$

$$\begin{aligned}\mathbb{E}[R_{i,j}|D_i=1] &= K\mathbb{E}[(I_0-D)^+|D_i=1]\\ &\quad + \sum_{k=i}^{n-1}\mathbb{E}[u_{k,I_k}e^{r(T-k\Delta t)}S(I_k,D_k)|D_i=1]\\ &= K\mathbb{E}[(I_0-D)^+] + \sum_{k=i+1}^{n-1}\mathbb{E}[u_{k,I_k}e^{r(T-k\Delta t)}S(I_k,D_k)]\\ &\quad + u_{i,I_i}e^{r(T-i\Delta t)}S(I_i,1).\end{aligned}$$

For $I_i > 0$, $S(I_i, 1) = 1$. Thus replacing I_i with j and using the notation that $E_{i,j} = \mathbb{E}[R_{i,j}]$, we get

$$\mathbb{E}[R_{i,j}|D_i=1] = E_{i+1,j-1} + u_{i,j}e^{r(T-i\Delta t)}.$$

Similarly, $\mathbb{E}[R_{i,j}|D_i = 0] = E_{i+1,j}$. Note that by assumption $\mathbb{P}(D_i = 0) = 1 - \lambda(u_{i,j})\Delta t$ and $\mathbb{P}(D_i = 1) = \lambda(u_{i,j})\Delta t$ so

$$\begin{aligned}E_{i,j} &= \lambda(u_{i,j})\Delta t(E_{i+1,j-1} + u_{i,j}e^{r(T-i\Delta t)}) \qquad (8)\\ &\quad + (1-\lambda(u_{i,j})\Delta t)E_{i+1,j}.\end{aligned}$$

Using the initial condition $E_{n,j} = jK$, we can iterate backward in time to find the expected remaining revenue at each (i, j); see Figure 2 for a visual representation of (8). However, the American put option has a payoff which may exceed the expected remaining revenue before the option's expiration, whereupon the risk-neutral retailer is certain to exercise. Then the expected remaining revenue is the immediate payout of exercising the option.

C. Payoff Policy

Given an American option contract, the payoff policy $K_{i,j}$ is the amount (in time T dollars) that the option pays out if executed at time period i when the inventory is j units. In each example below we assume the payoff policy is

$$K_{i,j} = jKe^{r(T-i\Delta t)}, \qquad (9)$$

which is the unit strike price K times the inventory level corrected for the future value at time T. Thus the expected remaining revenue propagates as a binomial tree in backward time with the following caveat: if the payoff $K_{i,j}$ is greater than $E_{i,j}$ then we replace $E_{i,j}$ with $K_{i,j}$. At these times and inventory levels, the retailer expects to earn more profit by exercising the option than by attempting to sell the rest of the inventory. This updates (8), which becomes

$$\begin{aligned}E_{i,j} = \max\{&K_{i,j}, \lambda(u_{i,j})\Delta t(E_{i+1,j-1} + u_{i,j}e^{r(T-i\Delta t)})\\ &+ (1-\lambda(u_{i,j})\Delta t)E_{i+1,j}\}. \qquad (10)\end{aligned}$$

We initialize (10) at time T via $E_{n,j} = K_{n,j}$, which is the option payoff at expiration; see discussion following (8).

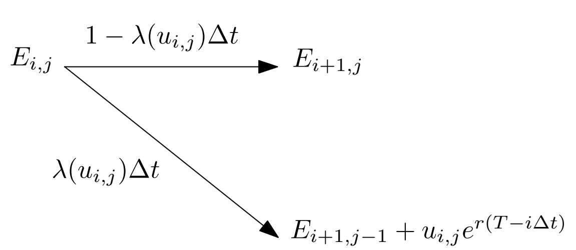


Fig. 2. Single period branch illustrating expected remaining revenue.

D. Optimal Stopping Times

Once we have the expected remaining revenues, we can derive the optimal stopping times at which the retailer should exercise the option: at an inventory level of j units, define

$$T(j) := \inf\{i \mid K_{i,j} \geq E_{i,j}\}. \qquad (11)$$

Note that since $E_{n,j} = K_{n,j}$, we have that $T(j) \leq T$ for all j. This is the optimal stopping time, and it represents the first instance in time that the payoff equals the expected remaining revenue, hence the moment that a risk-neutral retailer should exercise the option.

We must revise (3) to include the property that $P_{i+1,j} = 0$ if $i \geq T(j)$ since the option will have been exercised.

E. Optimal Pricing Policy

Although our option pricing solution will evaluate the situation in which the retailer chooses a suboptimal pricing policy, we assume by Bellman's principle of optimality that the retailer chooses prices to maximize the expected remaining revenue on each subinterval. We maximize expected remaining revenue by differentiating

$$\lambda(u_{i,j})\Delta t(E_{i+1,j-1}+u_{i,j}e^{r(T-i\Delta t)})+(1-\lambda(u_{i,j})\Delta t)E_{i+1,j}$$

with respect to $u_{i,j}$ and setting the derivative equal to zero so

$$\lambda'(u_{i,j})\left(\frac{E_{i+1,j}-E_{i+1,j-1}}{e^{r(T-i\Delta t)}} - u_{i,j}\right) = \lambda(u_{i,j}). \qquad (12)$$

Solving (12) for $u_{i,j}$ yields the optimal price $u^*_{i,j}$. Using the linear demand function $\lambda(u) = b - au$, $u^*_{i,j}$ satisfies

$$u^*_{i,j} = \frac{1}{2}\left(\frac{E_{i+1,j}-E_{i+1,j-1}}{e^{r(T-i\Delta t)}}\right) + \frac{b}{2a}. \qquad (13)$$

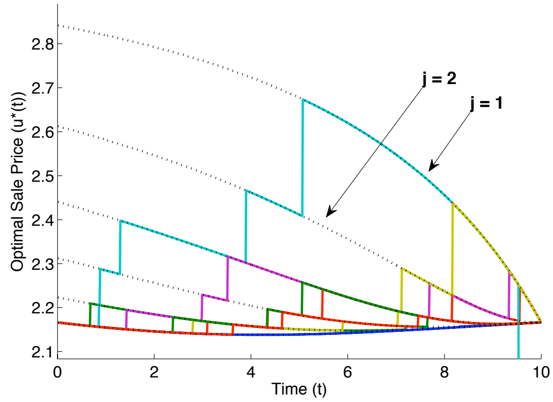
Using log-linear demand $\lambda(u) = bu^{-a}$, $a > 1$, we get

$$u^*_{i,j} = \frac{a}{a-1}\left(\frac{E_{i+1,j}-E_{i+1,j-1}}{e^{r(T-i\Delta t)}}\right). \qquad (14)$$

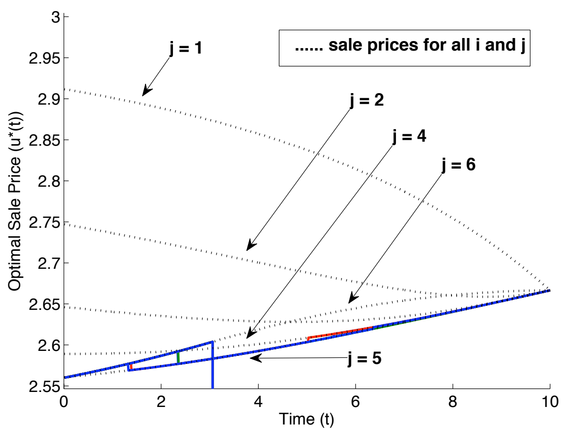
Thus the optimal pricing policy $\{u^*_{i,j}\}$ can be explicitly determined and incorporated into our algorithm with the assumption that the retailer is profit maximizing.

III. Pricing the Option

Recall that the risk-neutral price of the option is the expected value of the put option contract. We derive how to price the American retail put option contract by finding its expected payout, first by simulations and then by risk-neutral evaluation of the contract.



(a)



(b)

Fig. 3. Example of optimal price movements for $\lambda = 1 - .3u$, $I_0 = 6$, $T = 10$, and $r = .0133$. We have a strike price of (a) $K = 1$ thus generating a random walk throughout the entire contract period and (b) $K = 2$ usually producing an early exercise of the option.

A. Simulations

We used the inverse transform method to generate n independent and identically distributed Bernoulli random variables $X_1, \ldots, X_n$ with $p = \lambda(u^*(i\Delta t))\Delta t$. From this we were able to simulate our Poisson process incorporating the optimal stopping times $\{T(j)\}_{j=1}^{I_0}$. We used `rand`, MATLAB's random number generator, to generate numbers from a uniform distribution at each time step i. We dropped the inventory level by one if the random number was less than p. Otherwise we kept the inventory level constant. If we reached the optimal stopping time $T(j)$ for inventory j, then we calculated the payoff $K_{i,j}$ for that simulation. We set the option price to be the average payoff of multiple simulations.

In Figure 3, we show how the optimal price, $u^*(t)$, varies when the strike price K is (a) low, thus leading to sellout but not early execution and (b) high, producing a nearly deterministic path where early execution generally occurs. In the example shown in Figure 3(b), there were only 3 out of the 15 simulations that bypassed early execution (represented by the vertical line touching the x-axis at $t = 3$), selling just one or two goods before the end of the contract period. In Figure 3(a) the simulation on top sold out at $t = 9.5$, and every run moved extensively through the inventory levels as goods sold. We used only 15 runs here to get clear graphs.

At first, we used the inverse transform method to price the options, but these prices matched those given by our algorithm. We only need to compute the inventory probabilities and optimal exercise times and use these values in the closed form expression (see below) for the risk-neutral option price.

B. Closed Form Expression

Deferring to the risk-neutral valuation of the contract, the option value at time T is

$$p = \sum_{j=1}^{I_0} K_{T(j),j} P_{T(j),j}. \tag{15}$$

The payoff policy (9) simplifies (15). Then the option price in its present value is

$$p = K \sum_{j=1}^{I_0} j P_{T(j),j} e^{-r(T(j)\Delta t)}. \tag{16}$$

IV. ANALYSIS

In this section we explore some particulars with our option pricing algorithm. We first analyze how the option price changes as we vary parameters. Then we examine the graphs of the inventory probability levels of a situation where early execution can occur.

A. Option Price

In Figure 4(a), we see how the option price changes as we adjust the initial inventory. Clearly if the initial inventory is close to zero, then the probability of selling out is very high, thus the option price is low. In the case that the profit margins were also low, we would likely find the optimal stopping time to be $T(j) = 0$. Barring that, however, the relationship we see is as anticipated. For large initial inventories, we see that the option price has a linear relationship. Clearly if overstocked, the retailer is certain to execute and the payoff is, as expected, proportional to the inventory.

In Figure 4(b), we see an initially horizontal option price for short expiration times, followed by a sharp decline. For short contract periods, the retailer is certain to exercise the option. However, as the contract period lengthens, the retailer will be able to reasonably sell out, thus making the option inexpensive in the limit.

Finally, we consider a varying strike price in Figure 4(c). In the majority of our investigations we were able to achieve or negate early execution by adjusting our strike price. Indeed if the strike price is high, then the retailer is likely to execute early (usually when $t = 0$), but if it is low, the retailer would generally see through the end of the contract period. An interesting detail is the nearly vertical jump around $K = 2$ in our example. This suggests that the strike price has saturation points beyond which the change in strike price does not affect when to exercise the option. Notice that the saturation point on the right corresponds to an optimal stopping point at

$T(j) = 0$ whereas the saturation point on the left is unlikely to see early execution.

In our computations, it was generally difficult to find an optimal stopping time strictly between $T(j) = 0$ and $T(j) = T$. A likely reason for this is the strong dependence on strike price consistent with what we see in Figure 4(c). When far left of the saturation points, the American option is essentially a European option, with early exercise unlikely. From the standpoint of the option writer, it may still be marketable to sell an American option, as the retailer is willing to pay a premium for the extra flexibility in case of unexpected low demand. On the right of the saturation points, the American option is essentially a riskless and useless security since the retailer would instantly execute the option. Hence, we suggest that the American option is of most use in the alternative payoff examples in Section V.

B. Inventory Probabilities

A crucial step in our pricing process is the computing of the inventory probabilities $P_{i,j}$. As an example, in Figure 5, we explore the dynamics of the inventory probability curves for options with strike price $K = 1$ and $K = 2$. Each curve represents the probability that the inventory level is j at a given time $t = i\Delta t$. We note that the probability curve for the initial inventory starts at 1 and decreases monotonically while the other curves start at zero, hump in the middle, and then decay over time. The curve representing zero inventory is not shown here, but it grows to 1 if time on the graph were extended out, thus demonstrating as expected that the retailer would sell out if given an infinite amount of time.

Notice that the top curve in Figure 5(b) has a corner, corresponding to the option execution. This also has an effect on the other curves as the first can no longer feed into the second curve, meaning it is less likely to be in all states with less than 6 inventory.

V. ALTERNATIVE DEMAND RATES AND PAYOFF POLICIES

The American option is of the most value in a situation where the retailer is overstocked and would be better off liquidating inventory, rather than holding on until the end of the contract period and then liquidating. This might happen in situations where the underlying goods experience a significant change in demand, say at the end of a holiday season where there is a transition from a period of high demand to one of low demand. For example we might have an explicitly time-dependent stochastic demand rate

$$\lambda(u,t) = \begin{cases} b_1 u^{-a1} & t \in [0, t^*] \\ b_2 u^{-a2} & t \in (t^*, T], \end{cases}$$

where $a_1, a_2 > 1$. In this scenario, we still get (14), but a changes depending on time.

American options may also be useful in situations where the profit margins are smaller than the rate of interest, or where the retailer is experiencing cash-flow disruptions.

Retail options can also be used for inventories that perish or depreciate. Perishable inventories are those that drop significantly in value after a specified period of time and can usually only be sold for scrap at some salvage value; see for example [7]. Examples of perishable goods include magazines, bakery goods and produce items, but can also include electric power, theater tickets, and airline seats. By contrast, depreciable inventories decay in value over time. Consumer goods, such as cars, electronics, and computers fit this description. In either case, we can adjust the strike price of the option at the contract's expiration to reflect the good's future value or even throughout the contract period to model a varying rate of depreciation.

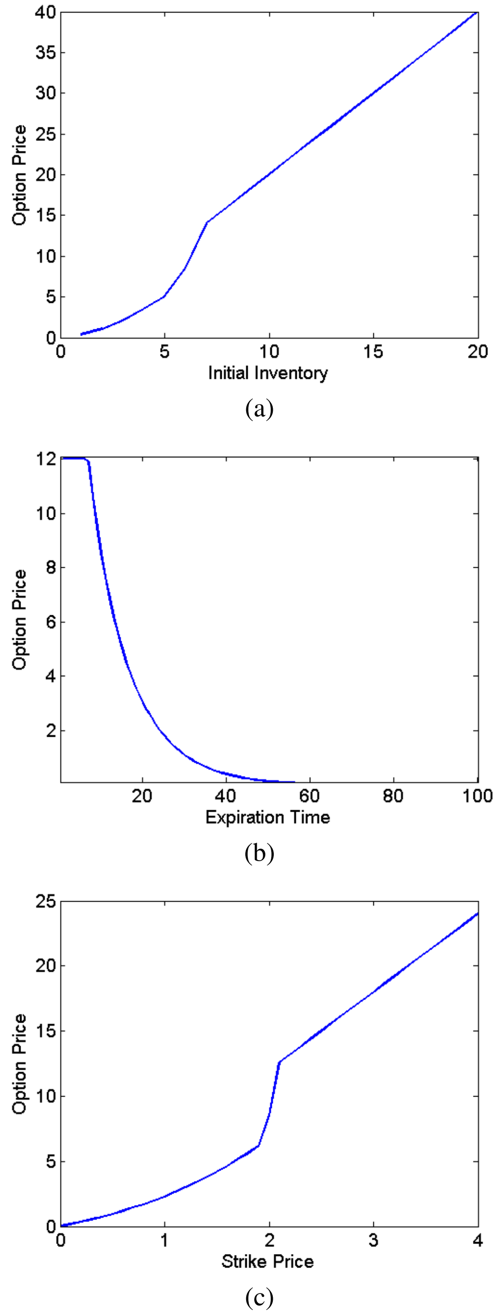


Fig. 4. Option price as we vary (a) the initial inventory, (b) the expiration time, and (c) the strike price.

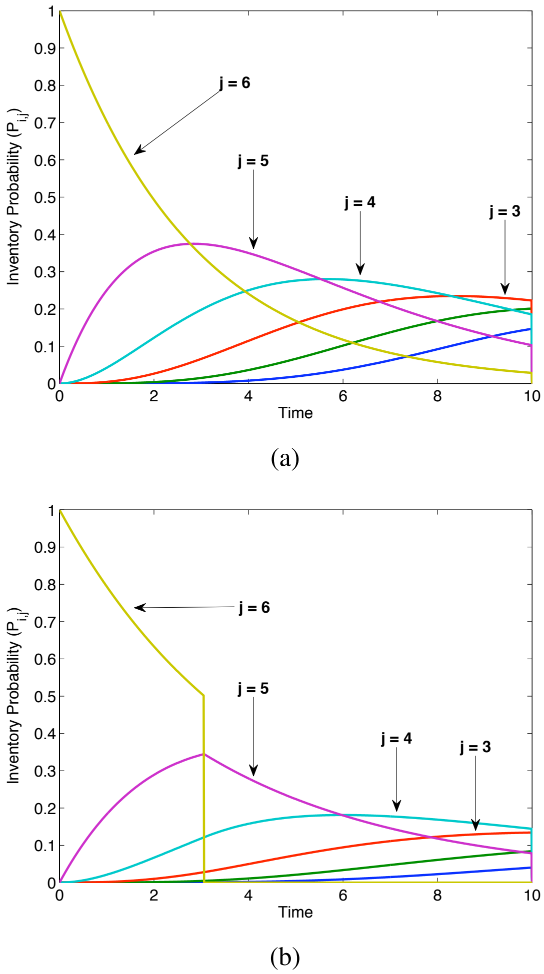


Fig. 5. Probability curves for different inventory levels. The parameters are $\lambda = 1 - .3u$, $I_0 = 6$, $T = 10$, $r = .0133$, and $\Delta t = .01$. In (a) we have $K = 1$ and in (b) $K = 2$. Note that the higher strike price results in the option having an early execution at $t = 3$.

One might also factor in storage and/or transportation costs to account for business expenses. Then seasonality of goods such as coats, swimwear, or umbrellas can be analyzed and the profits compared of storing the goods until next year versus liquidating the inventory now. We might model the demand rate as follows:

$$\lambda(u, t) = a - bu + \xi \cos(\gamma t + \phi).$$

Another variation could be dealer incentives, such as is common in the automobile industry. This could be modeled as a translation $u \rightarrow u + u_0$. However, if the incentive was time dependent, say if it was only for a short period of time, then the dynamics would change.

A final consideration might be blackout periods for exercising the option. We could set $K = 0$ during those times.

These are interesting areas for further investigation.

VI. DISCUSSION

We conclude with a brief discussion. There are many directions to take this portfolio view of inventory control theory. Several recent results exist for managing portfolios of options over multiple periods upon which we could expand; see for example [8]. We could also explore risk-management strategies including value-at-risk, cash-flow-at-risk, etc. Another direction is the use of more exotic options. Variations on floors, caps, Asian options, Bermuda options, etc., should all be considered for "retail possibilities."

Another area of interest is pricing an option when dealing with an incompetent retailer. Note that the option price proposed here requires that the retailer continually adjust prices to maximize profits; however, if the retailer isn't regulating them optimally, then the writer will be forced to buy more unsold inventory than necessary. While the existence of a Nash equilibrium should preclude such events theoretically, assuming symmetric information, market efficiency, etc., in practice, prices may be set badly. Room for such incompetence may need to be priced in the option.

Although the literature on retail options is fairly new, there has been an extensive "contracts" literature in inventory management going back a few decades, e.g., "buy-back" and "re-order" contracts; see [9] for a review. It would be interesting to study the relative merits of these contracts compared with those described here and elsewhere.

A final consideration is to explore whether it is better for the retailer to liquidate all inventory or just some when exercising the option early. Perhaps there are situations where the retailer wants to reduce inventory to a more profitable level. This is a current area of exploration for our group.

With the current global economic climate, there may be an opportunity for wholesalers, importers, and even governments to make use of options in a retail environment. The added security may reduce the reluctance with which a retailer is willing to hold more inventory with such uncertainty in the consumption, foreign exchange, and credit markets. Introducing American put options into retail environments may mitigate undesirable risk in inventory management.

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