Math 343 Lab 8: Eigenvalues

Objective

In this lab, we explore Matlab's eigenvalue solvers.

Eigenvalues and Eigenvectors

Recall that an $n \times n$ matrix A is said to have an eigenvalue λ if there exists a nonzero vector $\mathbf{v}$ such that

$$A\mathbf{v} = \lambda\mathbf{v}. \tag{1}$$

Any nonzero $\mathbf{v}$ satisfying (1) is called an eigenvector of A corresponding to the eigenvalue λ. The set of all eigenvalues A is called the spectrum of A.

In Matlab, we can find all the eigenvalues of a given matrix A via the `eig` command. Enter the following into Matlab:

```
>> A = [1 2 3;4 5 6;7 8 0]
```

```
>> eig(A)
```

More generally, we can use `eig` to find all of the eigenvalues as well as the corresponding eigenvectors. Do the following in Matlab:

```
>> [R,D] = eig(A)
```

Here, the column vectors of R are the eigenvectors of A corresponding to eigenvalues λ given in D. We note that in this case, the eigenvalues are given as diagonal entries of the matrix D. You can test this by showing that $A\mathbf{v} - \lambda\mathbf{v} = \mathbf{0}$ for each (λ,$\mathbf{v}$) pair. We do this in Matlab as follows:

```
>> A * R(:,1) - D(1,1) * R(:,1)
```

```
>> A * R(:,2) - D(2,2) * R(:,2)
```

```
>> A * R(:,3) - D(3,3) * R(:,3)
```

We remark that due to round-off error, we actually get really small numbers (on the order of `1e-14`) instead of getting zero.

Diagonalization

If the matrix R of eigenvectors has all linearly independent column vectors, then A can be converted into a diagonal matrix via the similarity transform $R^{-1}AR = D$. We verify this in Matlab as follows:

```
>> inv(R)*A*R
```

Note that this is the same as D.

Sparse Matrix Eigenvalue Solver

In most real-world applications, one usually does not need to know all of the eigenvalues, in fact, many applications require only the largest or smallest few eigenvalues. Matlab has an eigenvalue solver, called `eigs`, for sparse matrices that allows one to compute a few eigenvalues instead of computing all of them (which could take a very long time). Type `help eigs` in Matlab to get an idea of the options available. For example, if you wanted to compute the k eigenvalues closest to zero of a given sparse matrix A, you would enter `eigs(A,k,0)`

Assignment

Problem 1. *Using* `spTriSassy(n)` *from Lab 4, find the smallest eigenvalue λ for various values of n. What does $\lambda * n^2$ look like in the limit as n gets larger? Take n to the computational limit of your computer, that is, to the point where the run-time exceeds a reasonable period of time (say a few minutes).*