**Math 320 Midterm 2 Study Guide**

## 1. General information

- The exam will cover all sections from chapters 5–6 that were covered in class, as well as sections 7.1 and 7.2.
- The exam will be in the testing center from Monday November 27 through Wednesday November 29, with a late day on Wednesday starting at 2 PM. Check the testing center hours and lines, and be sure to give yourself enough time to finish.
- Books and notes will not be allowed. Testing center calculators will be used.
- This study guide is not exhaustive. Not appearing on the study guide does not mean that something will not appear on the exam.

## 2. Definitions

Know the definitions discussed in the book and in class, including:

(1) Sample space
(2) Power set of a set $S$
(3) Mutually exclusive and collectively exhaustive
(4) Discrete probability measure
(5) Equally likely outcomes
(6) $P(E|F)$, Independence of events
(7) Random variables (discrete and continuous and multivariate)
(8) Preimage of a set $S \subset B$ for a function $f : A \to B$
(9) Probability mass function
(10) Expectation/expected value of a random variable, variance
(11) Independence of random variables
(12) Indicator random variable
(13) cdf, pdf of a random variable
(14) Marginal pmf or pdf
(15) Covariance, covariance matrix
(16) Statistic, estimator, estimate
(17) Sample mean and variance estimators
(18) Biased and unbiased estimators
(19) Unbiased sample variance
(20) Likelihood of a parameter, maximum likelihood estimate
(21) Prior, posterior distributions
(22) Maximum a posteriori estimate
(23) Conjugacy
(24) Importance, inversion, rejection sampling

Be able to produce examples and non-examples for these definitions.

## 3. Theorems you should know and be able to state and be able to use

Be sure to know the full statement of each theorem, including all hypotheses.

(1) Chain Rule
(2) Law of Total Probability
(3) Bayes' Rule

(4) Expected value is linear (Theorem 5.4.14)
(5) Law of the Unconscious Statistician
(6) Theorems computing variance (5.4.20, 5.4.22, 5.4.23)
(7) Markov's and Chebyshev's inequalities
(8) Weak Law of Large Numbers
(9) Central Limit Theorem
(10) Monte Carlo estimate for integrals (uniform and nonuniform distributions)

## 4. Distributions you should know and be able to use

When appropriate, you should know the expected value, variance, pmf, or pdf of the Bernoulli, Binomial, Poisson, Uniform, Normal, Gamma, Beta, Multinomial, and Multivariate normal distributions, and know examples of when they may apply to real-world events.

## 5. Sample problems

(1) A fair 20-sided die is rolled 5 times. What is the probability that a prime is rolled exactly twice? at least twice?
(2) If you test positive for a disease that affects one in every $10^5$ people and the test is $99\%$ accurate, what is the probability you actually have the disease, absent any other information?
(3) Assume that the probability that a child is male (or female) is 50%. If you know that three of a family's four children are girls, what is the probability that the fourth is a girl? If you know that three of the four, including the oldest child, are girls, what is the probability that the fourth is a girl?
(4) A jar contains $m+n$ chips, with numbers $1, 2, \ldots, n+m$. A set of size $n$ is taken out of the jar. If we let $X$ denote the number of chips drawn which have numbers that are larger than all the numbers of the chips remaining in the jar, compute the probability mass function of $X$.
(5) The time in hours required to repair a machine is an exponentially distributed random variable with parameter $b = 1/2$. What is the probability that a repair time exceeds 2 hours? (The exponential distribution is a gamma distribution with $a = 1$.)
(6) For a sample $X_1, X_2$ from a distribution with mean $\mu$, show that $\tilde{\mu} = (3X_1 + 4X_2)/7$ is an unbiased estimator for $\mu$.
(7) A fair standard 6-sided die has been rolled 6 times. Use the Weak Law of Large Numbers to find an upper bound for the probability that the sum will not be 20, 21, or 22.
(8) Use the Central Limit Theorem to write an integral of the form $\frac{1}{\sqrt{2\pi}\sigma}\int_a^b e^{-(x-\mu)^2/2\sigma^2}dx$ that estimates the probability that the sum in the previous problem is 20, 21, or 22.
(9) Given that a coin with probability $p$ of coming up heads is flipped 4 times with results $H, H, H, T$, we have no prior reason to believe the coin is fair and start with a uniform prior Beta(1, 1) for $p$. Use the results of the 4 coin flips to deduce the Bayesian posterior probability distribution for $p$.
(10) Your friend tells you, over the phone, that he will roll a die four times. You do not know whether he is rolling a 6-sided die or a 20-sided die, but you think that both options are equally likely. He rolls the die four times, and tells you that he got a

result of 6 or lower each time. What is the posterior probability that he rolled a 20-sided die?

(11) Given a distribution $D$ with pdf equal to $f(x) = (1+x)^{-2}$ on $[0, \infty)$, explain how to use a sample from the uniform distribution on $[0, 1)$ to obtain a sample from $D$.

(12) If $f_X(t)$ is the pdf for the standard normal distribution, write $\mathbb{E}\left[\frac{\mathbb{1}_{[1,1+\ln(2)]}(t)e^t}{f_X(t)}\right]$ as an integral and compute its value. If you sample from the normal distribution 10 times, how would you compute the standard error when using this sample to compute the value of the integral? Explain.