TIMED

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NS

Math 303 (Sections 1 and 2)

QUIZ I

Name: ______________________________

Jan 18-Jan 19, 2018

Answer all questions and show all your work carefully. You should write your answers in the designated spaces (use the back of he page only if you need more space). Scratch paper can be used, but the work on scratch paper will not be graded. Basic scientific calculators (no graphic or symbolic ones) are allowed. There is a time limit of two and half hours.

When I say do your best, I mean your very best. You are capable of so much more.
Pres. Gordon B. Hinckley

Prof. Vladimir Soloviev

Prof. Vianey Villamizar

Problem No.	Points
1.-)	
2.-)	
Total	

MATH 303 QUIZ I

1. a) Consider the initial value problem

$$y' = (1-2x)/y, \quad y(1) = -2$$

i) **(5 points)** Find its solution in explicit form

ii) **(5 points)** Determine the interval in which the solution is defined (interval of validity).

i) $yy' = 1-2x$

$\int dx: \; \frac{y^2}{2} = x - x^2 + C$

$\Rightarrow \; y = \pm\sqrt{2x-2x^2+C}$

Using I.C.

$-2 = \pm\sqrt{2-2+C}$

$\Rightarrow \boxed{C=4}$

and

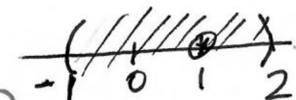
$\boxed{y(x) = -\sqrt{2x-2x^2+4}}$

ii) Interval of Validity:

$2x^2-2x-4 < 0$ (to avoid infinite tang. slope) >0

$x^2-x-2<0$

$(x-2)(x+1)<0$



$\Rightarrow$ Int. of Validity: $(-1, 2)$.

b) **(10 points)** Find the solution for the initial value problem

$$t^3y' + 4t^2y = e^{-t}, \quad y(-1) = 0 \qquad t<0.$$

In standard form:

$y' + \frac{4}{t}y = t^{-3}e^{-t}$

$\mu(t) = e^{4\int \frac{1}{t}dt} = e^{4\ln|t|} = |t|^4 = t^4$

$(yt^4)' = t^4(t^{-3}e^{-t}) = te^{-t}$

$\int dt: \; y(t)t^4 = \int te^{-t}dt = -te^{-t} - e^{-t} + C$

$\Rightarrow \; y(t) = \frac{-e^{-t}}{t^3} - \frac{e^{-t}}{t^4} + \frac{C}{t^4}$

$0 = y(-1) = \frac{-e^{-1}}{-1} - e^{-1} + C$

$= +\cancel{e^{-1}} - \cancel{e^{-1}} + C$

Then, $C=0$

and

$\boxed{y(t) = -e^{-t}(t^{-3}+t^{-4})}$

or

$\boxed{y(t) = -e^{-t}t^{-4}(t+1)}$

or

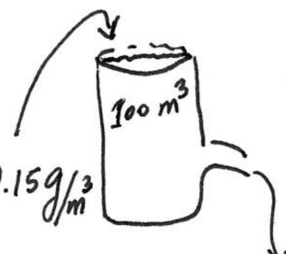
$\boxed{y(t) = \frac{-te^{-t}-e^{t}}{t^4}}$

2.) A tank initially contains 100 m^3 of water containing a pollutant at a concentration of 0.6 g/m^3. Water containing 0.15 g/m^3 of pollutant is pumped into the well-mixed tank at a rate of 5 m^3/s. The mixture flows out of the tank at the same rate.

a) (~~10~~ 9 **points**) Determine the amount and concentration of the pollutant in the tank as a function of time. Graph the result for the amount of pollutant in the tank as a function of time t.

b) (~~5~~ 1 **points**) Determine the amount of pollutant in the tank at time $t = 20$.

a)



$C_{in} = 0.15 g/m^3$

$r_{in} = 5 m^3/s$

$Q(t)$: Amount of pollutant at t seconds.

$Q(0) = 0.6 \, g/m^3 * 100\text{-}m^3 = 60 g.$ (Initial cond.)

$C_{out} = \dfrac{Q(t)\, g}{100 \, m^3}$, $r_{out} = 5 \, m^3/s.$

Conservation Law:

$$\frac{dQ}{dt}(t) = C_{in} r_{in} - C_{out} r_{out}$$

or

$$\boxed{\frac{dQ(t)}{dt} = (0.15)5 - 5\frac{Q(t)}{100}} \quad \boxed{\frac{dQ}{dt} = 0.75 - 0.05Q(t)}$$

or

$$\boxed{\frac{dQ}{dt}(t) = \frac{5}{100}(15 - Q(t))}$$

Then in differential form:

$$\frac{dQ}{Q - 15} = -\frac{1}{20} dt$$

thus,

$\int$: $\ln|Q - 15| = -\dfrac{t}{20} + C$

$e^{()}$: $Q - 15 = \tilde{C} e^{-t/20} \Rightarrow \boxed{Q(t) = 15 + \tilde{C} e^{-t/20}}$

Using I.C. $Q(0) = 60 \Rightarrow 60 = 15 + \tilde{C} \Rightarrow \boxed{\tilde{C} = 45}$

Substitution leads to

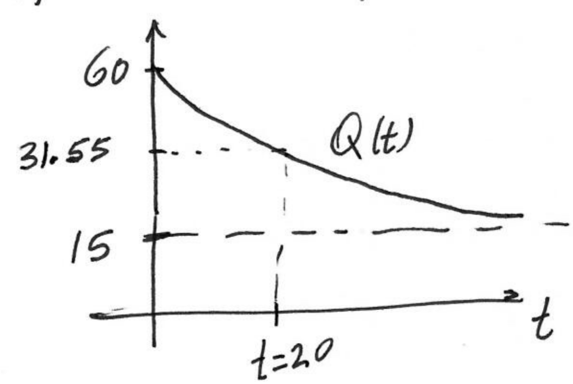
$$\boxed{Q(t) = 15 + 45\, e^{-t/20}}$$

Concentration: $C(t) = \dfrac{Q(t)\ g}{100\ m^3}$

or

$$\boxed{C(t) = 0.15 + 0.45\, e^{-t/20}}$$

Graph of amount of pollutant:



b) At $t = 20$

$$Q(20) = 15 + 45\, e^{-20/20} = 15 + 45\, e^{-1} = 15\left(1 + \frac{3}{e}\right)$$

or

$$\boxed{Q(20) = 15\left(1 + \frac{3}{e}\right) \simeq 31.55}$$