Homework 3, due September 13

(1) Compute $\gcd(455, 1235)$ and $\gcd(2415, 6909)$ by hand. Find integers x, y, z, w such that $455x + 1235y = 65$ and $2415z + 6909w = 63$.
(2) The Fibonacci numbers are given by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$. True or false: Any two consecutive Fibonacci numbers are relatively prime. If true, prove it. If false, give a counterexample.
(3) Let a and b be integers. Using the definition of congruence modulo n, prove that $a \equiv b$ (mod n) if and only if $a^2 + b^2 \equiv 2ab$ (mod n^2).