Applications Review

Fall04 (o) The centroid of the region $0 \le y \le x^2,\ -1 \le x \le 1$ is located at _____

W06 6. The definite integral $\int_0^3 2\pi x\sqrt{x^2+1}\,dx$ represents the volume of the solid of revolution generated by revolving the curve $y =$ _____, $x \in [$ ____, ____$]$, about the y-axis.

F07 a. Let $y = f(x)$ for $x \in [a, b]$. Give the formula for the length of the curve formed by the graph of this function. _____

F04 5. A ball of radius 5 has a hole of radius 2 drilled completely through it such that the axis of the hole is a diameter of the ball. What is the volume of what is left?

(a) $7\sqrt{21}\pi$

(b) $21\sqrt{21}\pi$

(c) $\frac{116}{3}\sqrt{29}\pi$

(d) $28\sqrt{21}\pi$

(e) $14\sqrt{21}\pi$

(f) $28\sqrt{21}\pi^2$

(g) None of the above

9. Which of the following integrals represents the surface area of the surface generated by revolving the arc $y = \sin x,\ 0 \le x \le \pi$ about the line $y = 2$.

(a) $\int_0^\pi 2\pi(1-\cos x)\sqrt{1+\sin^2 x}\,dx$

(b) $\int_0^\pi 2\pi(1-\sin x)\sqrt{1+\cos^2 x}\,dx$

(c) $\int_0^\pi 2\pi(2-\cos x)\sqrt{1+\sin^2 x}\,dx$

(d) $\int_0^\pi 2\pi(2-\sin x)\sqrt{1+\cos^2 x}\,dx$

(e) $\int_0^\pi 2\pi(1-\cos x)\sqrt{1+\cos^2 x}\,dx$

(f) $\int_0^\pi 2\pi(1-\sin x)\sqrt{1+\cos^2 x}\,dx$

(g) $\int_0^\pi 2\pi(2-\cos x)\sqrt{1+\cos^2 x}\,dx$

(h) $\int_0^\pi 2\pi(2-\sin x)\sqrt{1+\sin^2 x}\,dx$

F05 6. The length of the curve $y = \cosh x$ from $x = 0$ to $x = 1$ is

(a) $\sinh 1$

(b) $\cosh 1$

(c) $\cosh^2 1 - \cosh^2 0$

(d) 1

(e) ∞

(f) a real number in (0,1)

(g) Imaginary

(h) None of these

W06 8. The area of the surface of revolution generated by revolving the curve $y = \sin x, 0 \le x \le \pi$, about the line $y = -1$ is

(a) $\int_0^{\pi} 2\pi \sin x\sqrt{1+\cos^2 x}\, dx$ (b) $\int_0^{\pi} 2\pi \sin x\sqrt{1+\sin^2 x}\, dx$

(c) $\int_0^{\pi} 2\pi(1+\sin x)\sqrt{1+\cos^2 x}\, dx$ (d) $\int_0^{\pi} 2\pi(1-\sin x)\sqrt{1+\sin^2 x}\, dx$

(e) $\int_0^{\pi} 2\pi(\sin x-1)\sqrt{1+\sin^2 x}\, dx$ (f) $\int_0^{2\pi} \pi \sin x\sqrt{1+\cos^2 x}\, dx$

(g) $\int_0^{2\pi} \pi(1+\sin x)\sqrt{1+\sin^2 x}\, dx$ (h) none of these

10. Region R lies in the first quadrant, has area 6, and has centroid (4, 7). What is the volume of the solid generated by revolving R about the line $x = -1$?

(a) 36π (b) 42π (c) 48π (d) 54π (e) 60π (f) 72π (g) 84π

(h) 96π (i) none of these

F06 2. Which of the following integrals represents the surface area of the surface generated by revolving the curve $y = \tan x$, $0 \le x \le \pi/4$, about the line $y = -2$?

(a) $\int_0^{\pi/4} \pi(\tan x + 2)\sqrt{1+\sec^2 x}\, dx$

(b) $\int_0^{\pi/4} 2\pi(\tan x + 2)\sqrt{1+\sec^2 x}\, dx$

(c) $\int_0^{\pi/4} \pi(\tan x + 2)\sqrt{1+\sec^4 x}\, dx$

(d) $\int_0^{\pi/4} 2\pi(\tan x + 2)\sqrt{1+\sec^4 x}\, dx$

(e) $\int_0^{\pi/4} \pi(\tan x - 2)\sqrt{1+\sec^4 x}\, dx$

(f) $\int_0^{\pi/4} 2\pi(\tan x - 2)\sqrt{1+\sec^2 x}\, dx$

(g) $\int_0^{\pi/4} \pi(\tan x - 2)\sqrt{1+\sec^2 x}\, dx$

(h) $\int_0^{\pi/4} 2\pi(\tan x - 2)\sqrt{1+\sec^4 x}\, dx$

(i) None of the above

4. Consider the region R that is the portion of the circle $x^2+y^2 = 1$ that lies in the first quadrant. What is the volume of the solid generated by revolving R about the line $x + y = 2$?

(a) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi^2}{2}$ (g) $\frac{\pi^2\sqrt{2}}{3}$

(b) $\frac{\pi}{2}$ (e) $\frac{\pi^2}{3\sqrt{2}}$ (h) $\frac{\pi^2}{2\sqrt{2}}$

(c) $\frac{\pi\sqrt{2}}{3}$ (f) $\frac{\pi^2}{4}$ (i) None of the above

F07 7. Find the area between $y = 2x$ and $y = x^2$ for $x \in [-1, 3]$.

a) $\frac{8}{3}$ b) $\frac{4}{3}$ c) $-\frac{4}{3}$

d) 3 e) 4 f) None of the above

8. Let A denote the region between the graphs of $y = \cos(x) + 1$ and the line $y = 1$ for $x \in [0, \pi/2]$. A solid is obtained by revolving A about the x axis. find the volume of the solid.

a) $\frac{\pi^2}{4} + 2\pi$ b) $\frac{\pi^2}{2} + 2\pi$ c) $\frac{\pi^2}{2}$

d) $\frac{\pi^2}{4}$ e) $\frac{\pi^2}{4} - 1$ f) None of these

9. Which of the following integrals represents the surface area of the surface generated by revolving the curve $y = e^{2x}$ for $x \in [0, 1]$ about the line $y = -1$?

a) $\int_0^1 2\pi \left(e^{2x} - 1\right) \sqrt{1 + 4e^{4x}}\, dx$ b) $\int_0^1 \left(e^{2x} + 1\right) dx$ c) $\int_0^1 2\pi \left(e^{2x} + 1\right) \left(1 + 2e^{2x}\right)$

d) $\int_0^1 2\pi \left(e^{2x} + 1\right) \sqrt{1 + e^{4x}}\, dx$ e) $\int_0^1 \left(e^{4x} + 1\right) dx$ f) None of the above.

F04 12. Find the length of the curve $f(x) = \ln(\cos x)$ for $0 \leq x \leq \pi/4$.

13. Use the specified method to find the volume of the of the solid of revolution formed when the region bounded by $y = x^2$ and $y = x$ is revolved

(a) about the x- axis (discs)

(b) about the y-axis (shells)

14. A solid of uniform density has as its base the unit circle in the plane, and cross sections of the solid perpendicular to the x-axis are equilateral triangles. Find the mass of the solid.

15. Suppose Achilles can run 40 times as fast as the tortoise and that the tortoise has a lead of 100 paces at the beginning of a race. How far will Achilles run before he overtakes the tortoise?

F05 12. The region bounded by $y = x$ and $y = 2x^2$ is revolved about the **y-axis** ; find the volume of the solid generated.

13. Find the area of the surface of revolution generated by revolving the curve $y = \sqrt{x}$, $0 \le x \le 4$, about the x-axis.

14. Find the centroid of the region bounded by the curves

$$y = \sqrt{1+x^2}, \quad x = 1 \quad \text{and} \quad y = 1 + x.$$

Express you answer in terms of unevaluated integrals. (Note: You should simplify the integrands as much as possible.)

15. If a region in the first quadrant, with area 10π and centroid at the point $(1, 12)$, is revolved around the line $x = -5$, find the resulting volume of revolution.

W06 15. Find the length of the curve $y = \ln(\sin x), x \in [\frac{1}{6}\pi, \frac{1}{4}\pi]$.

17. A swimming pool has a circular window of radius 1.2 meters in a side wall. When the water in the pool exactly covers the lower half of the window, what is the force of the water pressure on the window? (Assume that the side wall is vertical; give your answer in terms of the weight-density w of the water.)

F06 11. Find the length of the graph of $y = \dfrac{1}{4}x^2 - \dfrac{1}{2}\ln x$, on the interval $1 \le x \le 2$.

12. Find the centroid of the region that lies within the first quadrant and is bounded above by $y = 1 - x^2$.

15. Find the mass of the circular region $x^2 + y^2 \le 1$, whose density at each point is twice the distance from the point to the origin.

F07 10 (8 points) A ball of radius 5 has a hole of radius 3 drilled completely through it such that the axis of the hole is a diameter of the ball. What is the volume of what is left?

11 (8 points) A spherical tank having radius 10 feet is filled with a fluid which weighs 100 pounds per cubic foot. This tank is half full. Find the work in foot pounds needed to pump the fluid out of a hole in the top of the tank.

13. (6 points) The economic trickle-down effect is based on the idea that injection of money into the economy reaches far beyond the initial receiver of funds. Say that one person receives a dollar and spends 4/5 of it. The person who receives that portion spends 4/5 of it in turn and so on and on "forever." The total cash flow from the first dollar is the sum of all these passed-on funds including the first dollar. What is that sum?

15 (8 points) A cube with 2 foot long sides is sitting on the bottom of an aquarium in which the water is 5 feet deep. Find the hydrostatic force on one of the sides of the cube. Use 62.5 pounds per cubic foot as the weight density of water.