Sequences and Series

F04 (m) The radius of convergence of $\sum_{n=0}^{\infty} x^n$ is ______________

(p) Indicate which convergence test one would use in determining the convergence/ divergence of

i. $\sum \frac{1}{n^2}$ ______________________________

ii. $\sum \frac{1}{\sqrt{n}}$ ______________________________

iii. $\sum \frac{1}{n^2+n}$ ______________________________

iv. $\sum \frac{1}{n^2-n}$ ______________________________

v. $\sum \frac{(-1)^n}{\sqrt{n}}$ ______________________________

vi. $\sum \left(\frac{1}{n}\right)^n$ ______________________________

(q) The MacLaurin series for $\cos x$ is ______________________________.
(Write at least 5 nonzero terms.)

F05 (i) Give the limit of the sequence $\left\{\left(1-\frac{1}{n}\right)^n\right\}$ as $n \to \infty$ if it is convergent, otherwise write DIVERGENT.

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(1) Let State the $(2n)$-th term of the MacLaurin series for $\dfrac{\sin x}{x}$

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(2) True/False: Write T if statement always holds, F otherwise.
Let $\sum a_n = \sum_{n=1}^{\infty} a_n$ be an arbitrary series.

(a) ____ If $\{a_n\}$ is a positive decreasing sequence then $\sum(-1)^n a_n$ converges

(b) ____ If $\sum a_n$ converges then $a_n \to 0$

(c) ____ If the partial sums of $\sum a_n$ are bounded, then $\sum a_n$ converges

W06 4. The Taylor series for e^{2x}, centered at 0, is ____________________

F06 (e) The radius of convergence of $\sum_{n=0}^{\infty} 3^n x^n$ is ____________

(g) The series $x^2 - \dfrac{x^4}{3!} + \dfrac{x^6}{5!} - \dfrac{x^8}{7!} + \ldots$ is the MacLaurin series for the function ____________

(i) The series $2 - \dfrac{2}{3} + \dfrac{2}{9} - \dfrac{2}{27} + \ldots$ converges to ____________

F07 (e) What does the ratio test predict with regard to the convergence of the series $\sum_{n=1}^{\infty} \dfrac{n^2}{n^4+1}$? ________________

(f.) What is the radius of convergence of the series $\sum_{n=1}^{\infty} \dfrac{4^n}{n} x^n$? ________________

F04 3. The sum $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

(I) $\int_1^{\infty} x^{-p}\, dx$ converges

(II) $p > 1$

(III) $p \leq 1$

(IV) $p \geq 1$

(V) $p < 1$

Which of the above are true?

(a) (I),(II)

(b) (II) only

(c) (I) and (II)

(d) (III) only

(e) (I) and (IV)

(f) (IV) only

(g) (I) and (V)

(h) (V) only

(i) None of the above

6. Find the sum of the series $\sum_{n=1}^{\infty} n\,(-1)^n \left(\frac{1}{2}\right)^n$.

(a) $-\frac{13}{18}$

(b) $-\frac{20}{9}$

(c) $\frac{7}{9}$

(d) $-\frac{2}{9}$

(e) $\frac{1}{9}$

(f) $-\frac{5}{9}$

(g) $\frac{5}{18}$

(h) None of the above

F05 4. The series $x^2 + x^4 + \frac{x^6}{2} + \frac{x^8}{6} + \cdots = \sum_{n=0}^{\infty} \frac{x^{(2n+2)}}{n!}$ converges to the function

(a) $\frac{x^2}{1+x^2}$

(b) $x^2 \tan^{-1} x$

(c) e^{x^2+2}

(d) $x^2 e^{x^2}$

(e) $x^2(\sin x^2 + \cos x^2)$

(f) $\sin x^2 + \cos x^2$

(g) None of these

8. The interval of convergence of the power series $\sum_{n=1}^{\infty} n^2 (5x-3)^n$ is

(a) $(-3/5, 3/5)$

(b) $(-5/3, 5/3)$

(c) $(0, 1)$

(d) $(-1, 1)$

(e) $(2/5, 4/5)$

(f) $(1/5, 1)$

(g) $(0, \infty)$

(h) $(-\infty, \infty)$

(i) None of the above

9. The coefficient of x^3 in the series expansion of $(1+x)^{1/4}$ is

(a) $\frac{1}{4^3} = \frac{1}{64}$

(b) $\frac{1}{4^3 3!} = \frac{1}{384}$

(c) $\frac{6}{4^3 3!} = \frac{1}{64}$

(d) $\frac{15}{4^3 3!} = \frac{5}{128}$

(e) $\frac{20}{4^3 3!} = \frac{5}{96}$

(f) $\frac{21}{4^3 3!} = \frac{7}{128}$

(g) $\frac{25}{4^3 3!} = \frac{25}{384}$

(h) $\frac{35}{4^3 3!} = \frac{35}{384}$

(i) None of the above

W06 11. Which best describes how the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ behaves?

(a) converges absolutely (b) converges conditionally, not absolutely (c) doesn't converge

(d) converges to infinity (e) converges to negative infinity

12. By using two non-zero terms of the Taylor series, the integral $\int_0^1 \sin(x^2)\, dx$ is approximately (to two digit accuracy)

(a) .96 (b) .43 (c) .31 (d) .29 (e) .12 (f) .03

F06 5. The series $\sum_{n=2}^{\infty} \frac{3^n}{n!}$ converges to

(a) $\ln 3$

(b) $\ln 2$

(c) $\ln(3) - 1$

(d) $\dfrac{3^{n+1}}{n+1}$

(e) ∞

(f) e^3

(g) $\cos 3$

(h) $e^3 - 4$

(i) 3^e

6. The interval of convergence of the power series $\sum_{n=1}^{\infty} n^2(7x-3)^n$ is

(a) $\left(-\frac{3}{7}, \frac{3}{7}\right)$

(b) $\left(-\frac{7}{3}, \frac{7}{3}\right)$

(c) $(-1, 1)$

(d) $(0, 1)$

(e) $\left(\frac{1}{7}, 1\right)$

(f) $\left(\frac{2}{7}, \frac{4}{7}\right)$

(g) $(0, \infty)$

(h) $(-\infty, \infty)$

(i) None of these

F07 4. Which of the following series has as its interval of convergence $(-\infty, 1]$?

a) $\sum x^2/n!$ b) $\sum e^{-n}x^{2n}$ c) $\sum \frac{(x-1)^n}{2^n}$

d) $\sum x^{n-1}/n^2$ e) $\sum (x+1)^{n+1}/n^n$ f) None of these

5. Determine whether the following series is convergent. If it is, find its sum.

$$\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$$

a) It is divergent. b) e c) $\frac{1}{e}$

d) $e - e^{1/2}$ e) $\ln(1)$ f) None of these.

6. If we differentiate the power series

$$\frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \frac{x^9}{9!} + \ldots$$

the resulting function is

a) e^{-x} b) $\cos 2x$ c) $\sin x$

d) only expressible as a series. e) $-(\cos x - 1)$ f) $\cosh x + 1$

F04 16. Write down the Taylor series for $f(x) = \dfrac{1+x}{1-x}$ about $x = 0$. You should include the nth term in the Taylor series. Hence find the fifth derivative of $f(x)$ at $x = 0$:

$$\frac{d^5}{dx^5}\left(\frac{1+x}{1-x}\right)\bigg|_{x=0}$$

F05 17. a) Determine the power series expansion of $\int \tan^{-1} x\, dx$.

b) Find first two nonzero terms of the Taylor series of $\ln(1 + \sin^2 x)$ at $x = \pi$. What is the remainder after these terms?

16. Determine whether each infinite series is absolutely convergent, conditionally convergent, or divergent. Give reasons for your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{3n+7}$

(b) $\displaystyle\sum_{n=1}^{\infty} (3^{-n} - 5^{-n})$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(2n)}$

WOL 20. Find the Taylor series for the function $f(x) = x^2 + 2x + 5$ centered at the point $x = 2$. Show your work.

21. For each of the following power series, determine what function it converges to, and find the interval of convergence (watch the end-points).

(a) $\sum_{n=0}^{\infty} nx^{n-1}$

(b) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n}}{n!}$

22. (a) Give the formal definition of the statement $\lim_{n\to\infty} a_n = L$.

(b) Determine the limit $\lim_{n\to\infty} \left(1 + \frac{\pi}{n}\right)^n$ (by any means).

F06 14. Use the first three non-zero terms of the MacLaurin series for e^{-x^2} to estimate the definite integral $\int_0^2 e^{-x^2}\, dx$. Write your answer as a fraction, if possible.

16. Find the sum of the power series $\sum_{n=1}^{\infty} nx^{n-1}$ (as a rational function of x).

17. Determine whether each of the following infinite series converges. State any convergence/divergence test you used.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

(b) $\sum_{n=1}^{\infty} \frac{e^n}{n^{30}+2^n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

F07 12. (6 points) Determine whether the following series converges and explain your answer.

$$\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$$

14. (6 points) Compute the first 4 nonzero terms of the MacLaurin series for the function $f(x) = \ln(x^2+1)$.