1. (10%) Find the length of the curve $y = \ln(\cos x)$, $0 \le x \le \pi/3$.

2. (10%) Show that the area S of a sphere or radius r is given by $S = 4\pi r^2$. Do this by rotating a half circle of radius r about the $x-$axis.
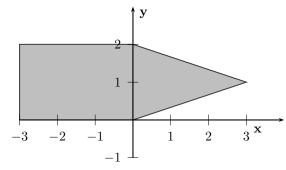
3. (15%) A large whale aquarium has a circular window at one end of radius 3 ft, The center of the window is 10 ft below the surface

   (a) Set up an integral to evaluate the force of water on the window. Use δ for the density of water.

   (b) Find the force on the window by using the theorem of Pappus. Leave your answer in terms of δ.

4. (10%) Find the centroid of the region under the curve $y = \sin x$ and above the x-axis for $0 \le x \le \pi$.

5. (15%) For $c > 0$, consider the function $f(t) = \begin{cases} 0 & \text{if } t < 0 \\ ce^{-ct} & \text{if } t \geq 0 \end{cases}$.

    (a) Show that f is a probability density function.

    (b) Find the mean μ in terms of c. (Show your work.)

    (c) Find a formula for the median M in terms of c. (Show your work.)

6. (10%) Find the centroid of the following system consisting of a rectangle and and an isosceles triangle.

7. (5%) Give the definition of $\lim_{n \to \infty} a_n = L$.

8. (15%) Evaluate the following limits if they exist. If the limit does not exist, so state.

(a) $\lim\limits_{n\to\infty} \sqrt{n}\sin\dfrac{\pi}{\sqrt{n}}$

(b) $\lim\limits_{n\to\infty} \sin\dfrac{\pi}{\sqrt{n}}$

(c) $\lim\limits_{n\to\infty} \cos n\pi$

9. (10%) Show $\lim\limits_{n\to\infty} r^n = 0$ if $0 < r < 1$. Explain all of your reasoning.

10. (Bonus) Show the sequence $\left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \cdots \right\}$ is bounded and increasing. Find the limit.

Hint: If the terms in the sequence are given by a_n, find a formula for a_{n+1} in terms of a_n. Show the sequence is increasing and bounded above by 2.