Math 113-008, Exam 4
Nov. 30–Dec. 3, 2007
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Name ______________________
Row______________________

Unless indicated, each problem is worth 5%.

1. (30%) Determine whether each infinite series is absolutely convergent, conditionally convergent, or divergent. Give reasons for your conclusion.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^{1.01}}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^3}}{\sqrt{k^5}+1}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{5 \cdot 8 \cdot 11 \cdot \cdots \cdot (3n+2)}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

   (f) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k^2+1}{2k^2+1}\right)^{2k}$

2. Find the coefficient of x^{1002} in the MacLaurin series for $x^2 \cos x^2$.

   (a) $\dfrac{-1}{499!}$ (b) $\dfrac{-1}{500!}$ (c) $\dfrac{-1}{999!}$ (d) $\dfrac{-1}{1000!}$ (e) $\dfrac{1}{499!}$ (f) $\dfrac{1}{500!}$ (g) $\dfrac{1}{999!}$ (h) $\dfrac{1}{1000!}$

3. Evaluate the sum of the geometric series $2 - \frac{1}{3} + \frac{1}{18} - \cdots$

(a) $\frac{12}{7}$ (b) $\frac{12}{5}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$ (e) $\frac{1}{6}$ (f) $-\frac{1}{6}$ (g) $\frac{5}{6}$ (h) 2

4. Find the sum $\sum_{k=0}^{\infty} \frac{2}{k^2 + 4k + 3}$

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$ (e) $\frac{3}{2}$ (f) $\frac{5}{2}$ (g) $\frac{7}{2}$ (h) $\sqrt{e}$

5. Evaluate the sum. $1 + 2x + 3x^2 + 4x^3 + \cdots = \sum_{n=0}^{\infty} (n+1)x^n$

(a) $\frac{1}{1-x}$ (b) $\frac{x}{1-x}$ (c) $\frac{1}{(1-x)^2}$ (d) $\frac{x}{(1-x)^2}$ (e) $\frac{1}{(1-x)^3}$ (f) $\frac{2}{(1-x)^3}$ (g) $\frac{x}{(1-x)^3}$ (h) $\frac{2x}{(1-x)^3}$

6. Evaluate the sum. $x^4 - \frac{x^6}{2!} + \frac{x^8}{4!} - \frac{x^{10}}{6!} + \cdots = \sum_{k=2}^{\infty} \frac{(-1)^k x^{2k}}{(2k-4)!}$

(a) $x^2 \cos x$ (b) $x^4 \cos x$ (c) $x^4 \cos x^2$ (d) $x^3 \sin x$

7. Evaluate the following limit: $\lim_{x \to 0} \frac{3 \tan^{-1} x - 3x + x^3}{x^5}$

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$ (e) $\frac{3}{5}$ (f) $-\frac{3}{5}$ (g) $\frac{1}{4}$ (h) $-\frac{1}{4}$

8. Find the the coefficient of x^7 in the power series expansion for the function $\sin^{-1} x$ or $\arcsin x$ expanded about $x = 0$.

(a) 0 (b) 1 (c) $\frac{5}{16}$ (d) $\frac{5}{112}$ (e) $\frac{15}{112}$

9. Find the radius of convergence. $\sum_{n=1}^{\infty} \frac{(-1)^n n^3 x^n}{2^n}$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\sqrt{2}$

10. Find the radius of convergence. $\sum_{n=1}^{\infty} \frac{x^{2n}}{n 4^n}$

a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\sqrt{2}$

11. Find the coefficient of x^4 in the MacLaurin series for $e^{x^2} \cos x$.

(a) 0 (b) $\frac{1}{6}$ (c) $-\frac{1}{6}$ (d) $\frac{1}{24}$ (e) $-\frac{1}{24}$

12. Find the fourth non-zero term for the Taylor series for $\sin x$ about $x = \pi/2$.

(a) $\dfrac{(x-\pi/2)^6}{6!}$

(b) $-\dfrac{(x-\pi/2)^6}{6!}$

(c) $\dfrac{(x-\pi/2)^8}{8!}$

(d) $-\dfrac{(x-\pi/2)^8}{8!}$

13. Find the Taylor series for e^{2x} about $x = 1$.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{e^2(x-1)^n}{2^n n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n e^2(x-1)^n}{n!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-1)^n}{e^2 n!}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{e^2(x+1)^n}{2^n n!}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n e^2(x+1)^n}{n!}$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x+1)^n}{e^2 n!}$

14. Find the Maclaurin series for $\dfrac{1}{x^2-1}$.

(a) $1 + x^2 + x^4 + x^6 + \cdots$

(b) $-1 - x^2 - x^4 - x^6 + \cdots$

(c) $1 - x^2 + x^4 - x^6 + \cdots$

(d) $-1 + x^2 - x^4 + x^6 + \cdots$

15. Find the Maclaurin series for xe^{-2x}

(a) $x - 2x^2 + \dfrac{2^2x^3}{2!} - \dfrac{2^3x^4}{3!} + \cdots$

(b) $x + 2x^2 + \dfrac{2^2x^3}{2!} + \dfrac{2^3x^4}{3!} + \cdots$

(c) $x^2 - 2x^3 + \dfrac{2^2x^4}{2!} - \dfrac{2^3x^5}{3!} + \cdots$

(d) $2x - 2^2x^2 + \dfrac{2^3x^3}{2!} - \dfrac{2^4x^4}{3!} + \cdots$

16. (bonus) If $f(x) = x^2 \cos x$, find the 100th derivative evaluated at zero; i.e., find $f^{(100)}(0)$.

1. (a) Converges by Alternating Series Test. Does not converge absolutely by the Integral Test. So the series converges conditionally.

   (b) Converges absolutely by Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^{1.005}}$.

   $\frac{\ln n}{n^{1.01}} = \frac{\ln n}{n^{.005}} \frac{1}{n^{1.005}} < \frac{1}{n^{1.005}}$ for large n.

   (c) Diverges by Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{k^{3/2}}{k^{5/2}} = \sum_{k=1}^{\infty} \frac{1}{k}$

   (d) Converges absolutely by Ratio Test.

   (e) Diverges by Test for Divergence or by Ratio Test.

   (f) Converges absolutely by the Root Test.

2. (f)
3. (a)
4. (e)
5. (c)
6. (b)
7. (e)
8. (d)
9. (c)
10. (c)
11. (d)
12. (b)
13. (b)
14. (d)
15. (a)
16. (bonus) -9900