Name: ______________________

Student ID: ______________________

Section: ______________________

Instructor: ______________________

Math 113 (Calculus 2)
Exam 2

13–19 February 2008

Instructions:

1. Work on scratch paper will not be graded.
2. For questions 2 to 6, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
3. Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
4. Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
5. Calculators are not allowed.

For Instructor use only.

#	Possible	Earned		#	Possible	Earned
1. a, b, c, d	16			2	8	
1. e, f, g	12			3	8	
1. h, i, j	12			4	8	
1. k, l, m	12			5	8	
1. n, o	8			6	8	
				Total	100	

Short Answer/Multiple Choice (60 points). Fill in the blank with the appropriate answer or circle the correct answer. You do not need to show your work.

1. (a) The integral $\int_1^e x \ln x \, dx$ is equal to

    i. $\frac{e^2-1}{2}$
    ii. e^2+1
    iii. $\frac{e^2+1}{2}$
    iv. $\frac{e^2+1}{4}$ Correct Answer
    v. $\frac{e^2-1}{4}$
    vi. e^2-1

   (b) Which of the following substitutions will best simplify the integral $\int \sqrt{3+2x-x^2}\, dx$

    i. $x = 1 - 2\sec\theta$
    ii. $x = \sqrt{3} + 2\cos\theta$
    iii. $x = \sqrt{3}\cos\theta$
    iv. $x = \sqrt{3} - 2\cos\theta$
    v. $x = \sqrt{3}\sin\theta$
    vi. $x = 1 + 2\sin\theta$ Correct Answer
    vii. $x = 2\sin\theta$

   (c) What substitution would you use in order to find the antiderivative $\int \sqrt{16+x^2}\, dx$?

   $x = 4\tan\theta$

   (d) What identity would you use in order to find the antiderivative $\int \sin(3x)\cos(2x)\, dx$?

   $\sin(3x)\cos(2x) = \dfrac{\sin(5x)+\sin(x)}{2}$

   (e) Does the integral $\int_0^\infty \frac{dx}{1+x^2}$ converge? If yes, give its value. $\pi/2$

   (f) Does the integral $\int_0^1 \frac{dx}{\sqrt[3]{x}}$ converge? If yes, give its value. 3/2

   (g) Does the integral $\int_1^\infty \frac{dx}{\sqrt[3]{x}}$ converge? If yes, give its value. diverges

   (h) Does the improper integral $\int_0^\infty \frac{dx}{e^x+1}$ converge (yes or no) yes

(i) The integral $\int_1^2 \frac{x^2+1}{x} dx$ equals $3/2 + \ln 2$

(j) The integral $\int_1^2 \ln x \, dx$ equals $2\ln 2 - 1$

(k) The integral $\int_0^{\pi} \cos^2 y \, dy$ equals $\pi/2$

(l) Give the integral definition of $\ln x$. $\ln x = \int_1^x \frac{dt}{t} \quad x > 0$

(m) The temperature in degrees Fahrenheit over a two-hour period is given by $T(t) = 70 + 5\sin(\pi t)$, $0 \le t \le 2$. Find the average temperature in degrees Fahrenheit.

70

(n) If $\sin\theta = x$, find $\sin(2\theta)$ in terms of x. $2x\sqrt{1-x^2}$

(o) The Mean Value Theorem for Integrals states that if f is a continuous function on $[a, b]$, then there exists a number c in $[a, b]$ such that

$f(c)(b-a) = \int_a^b f(x)dx$

Evaluate the following integrals (40 points). For problems 2 through 6 you must show all of your work. Write the final answer in the blank.

2. $\int \frac{x^2+x+1}{x^3+x} dx$

Use partial fractions decomposition.

$$\int \frac{x^2+x+1}{x^3+x} dx = \int \left(\frac{1}{x^2+1} + \frac{1}{x}\right) dx = \tan^{-1} x + \ln|x| + C$$

3. $\int_0^{\pi/6} \tan(2x)\sec^4(2x) \, dx$

$$\int_0^{\pi/6} \tan(2x)\sec^4(2x) \, dx = \frac{1}{2}\int_0^{\pi/6} \sec^3(2x)(2\sec(2x)\tan(2x))dx$$

$$u = \sec(2x), \quad du = 2\sec(2x)\tan(2x)dx, x = 0 \Rightarrow u = 1, \quad x = \pi/6 \Rightarrow u = 2$$

$$\frac{1}{2}\int_1^2 u^3 du = \frac{u^4}{8}\Big|_1^2 = \frac{15}{8}$$

4. $\displaystyle\int_0^2 x^3\sqrt{4-x^2}\,dx$

Use Trig Substitution $x = 2\sin\theta,\quad dx = 2\cos\theta d\theta,\quad x = 0 \Rightarrow \theta = 0,\quad x = 2 \Rightarrow \theta = \pi/2$

$$\int_0^{\pi/2} 8\sin^3\theta\sqrt{4(1-\sin^2\theta)}\;2\cos\theta d\theta = 32\int_0^{\pi/2}\sin^3\theta\cos^2\theta\;d\theta$$

$$= -32\int_0^{\pi/2}(1-\cos^2\theta)\cos^2\theta(-\sin\theta d\theta)$$

$$u = \cos\theta,\quad du = -\sin\theta d\theta,\quad \theta = 0 \Rightarrow u = 1,\quad \theta = \pi/2 \Rightarrow u = 0$$

$$\int_1^0 (u^2-u^4)du = \left(\frac{u^3}{3}-\frac{u^5}{5}\right)\Big|_1^0 = \frac{64}{15}$$

5. $\displaystyle\int_0^\infty \frac{x\arctan x}{(1+x^2)^2}\,dx$

Trig Sustitution $x = \tan\theta,\quad dx = \sec^2\theta d\theta,\quad x = 0 \Rightarrow \theta = 0,\quad x = \infty \Rightarrow \theta = \pi/2$

$$\int_0^{\pi/2}\frac{\theta\tan\theta}{(1+\tan^2\theta)^2}\sec^2\theta d\theta = \int_0^{\pi/2}\frac{\theta\tan\theta}{(\sec^2\theta)^2}\sec^2\theta d\theta = \int_0^{\pi/2}\frac{\theta\tan\theta}{\sec^2\theta}d\theta = \int_0^{\pi/2}\theta\sin\theta\cos\theta d\theta$$

Integration by Parts: $U = \theta,\quad dV = \sin\theta\cos\theta d\theta,\quad dU = d\theta,\quad V = \dfrac{\sin^2\theta}{2}$

$$= \theta\frac{\sin^2\theta}{2}\Big|_0^{\pi/2} - \int_0^{\pi/2}\frac{\sin^2\theta}{2}d\theta = \frac{\pi}{8}$$

6. $\displaystyle\int \sec^3 z\,dz$

Integration by Parts: $u = \sec z,\quad dv = \sec^2 z\;dz,\quad du = \sec z\tan z dx,\quad v = \tan z$

$$\int\sec^3 z\,dz = \sec z\tan z - \int\sec z\tan^2 z dz$$

$$= \sec z\tan z - \int\sec z(\sec^2 z - 1)dz$$

$$= \sec z\tan z - \int\sec^3 z dz + \int\sec z dz$$

Using the formula for $\displaystyle\int\sec z dz$ and solving for the required integral, we get

$$\int\sec^3 z\;dz = \frac{1}{2}(\sec z\tan z + \ln|\sec z + \tan z|) + C$$