Name: ______________________

Student ID: ______________________

Section: ______________________

Instructor: ______________________

Math 113 (Calculus 2)
Exam 3

7–11 March 2008

Instructions:

1. Work on scratch paper will not be graded.
2. For questions 4 to 8, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
3. Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
4. Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
5. Calculators are not allowed.

For Instructor use only.

#	Possible	Earned		#	Possible	Earned
1. a, b	10			5	10	
1. c, d	10			6	10	
2.	15			7	10	
3.	15			8	10	
4.	10			Total	100	

1-3 Short Answer Fill in the blank with the appropriate answer. You do not need to show your work.

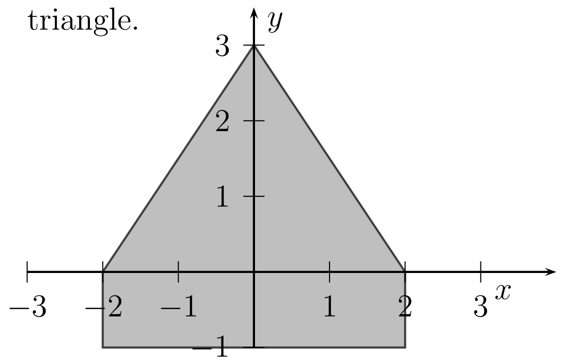
1. (a) Write an integral that represents the surface area when the curve $y = \tan x$, $0 \leq x \leq \pi/4$ is revolved about the line $y = -2$.

$$2\pi \int_0^{\pi/4} (\tan x + 2)\sqrt{1 + \sec^4}\ dx$$

(b) What is the hydrostatic force on a 2 feet by 2 feet square aquarium window whose top is 4 feet below the surface of the water if the density of water is 62.5 lbs/ft^3?

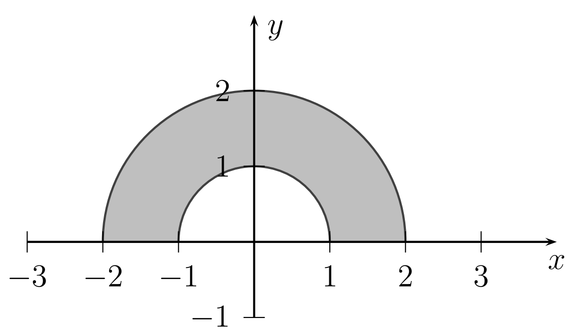
$5 \times 4 \times 62.5\text{lbs} = 1250\text{lbs}$

(c) Find the centroid of the following system consisting of a rectangle and and a triangle.



$(0, 2/5)$

(d) Find the centroid of the region between the two semicircles in the $x-y$ plane. You may use either Hint 1 or Hint 2. Hint 1: The area can be found as the difference of two areas. In a similar manner, the moment about the x-axis can be found as the difference of two moments. Hint 2: Use the Theorem of Pappus.



$(0, \dfrac{28}{9\pi})$

2. (a) Give the definition of $\lim_{n\to\infty} a_n = L$

Given $\epsilon > 0$, there is an integer N so that $n > N$ implies $|a_n - L| < \epsilon$

(b) A sequence $\{a_n\}$ is defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$ for $n \geq 1$. Assuming that the sequence is convergent, find its limit.

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(c) For what values of r is it true that $\lim_{n\to\infty} r^n = 0$?

$-1 < r < 1$

3. Evaluate the following limits if they exist. If the limit does not exist, so state.

(a) $\lim_{n\to\infty} \frac{n!}{n^n} =$ 0

(b) $\lim_{n\to\infty} (-1)^n \cos \frac{1}{n} =$ Does not exist.

(c) $\lim_{n\to\infty} n \sin \frac{\pi}{n} =$ π

For problems 4 through 8 you must show all of your work. Write the final answer in the blank.

4. Consider the curve $y = \frac{x^4}{16} + \frac{1}{2x^2}$, $1 \leq x \leq 2$.

(a) Set up and simplify an integral for the length of the curve. Do not evaluate.

$$\frac{dy}{dx} = \frac{x^3}{4} - \frac{1}{x^3}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{x^6}{16} - \frac{1}{2} + \frac{1}{x^6}} = \sqrt{\frac{x^6}{16} + \frac{1}{2} + \frac{1}{x^6}} = \sqrt{\left(\frac{x^3}{4} + \frac{1}{x^3}\right)^2} = \frac{x^3}{4} + \frac{1}{x^3}$$

Answer: $\int_1^2 \left(\frac{x^3}{4} + \frac{1}{x^3}\right) dx$

(b) Set up and simplify an integral for the area of the surface obtained by rotating the curve in part (a) about the y-axis. Do not evaluate.

Answer: $2\pi \int_1^2 x \left(\frac{x^3}{4} + \frac{1}{x^3}\right) dx$

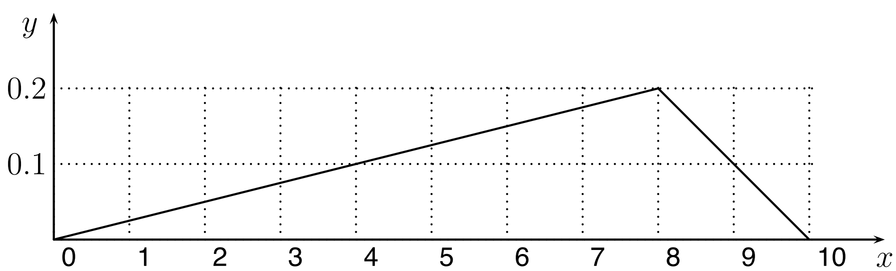
5. A swimming pool has a circular window of radius 1.2 meters in a side wall. The water level in the pool is exactly 0.2 meters above the horizontal diameter. Set up (do not evaluate) an integral that represents the force of the water on the window. (Assume that the side wall is vertical; give your answer in terms of the weight-density w of the water.)

Equation of circle $x^2 + y^2 = 1.44$ Solving for x we get $x = \pm\sqrt{1.44 - y^2}$ When $-1.2 \leq y \leq 0.2$, the depth at y is $0.2 - y$, and the width of a rectangle at y is given by $2\sqrt{1.44 - y^2}$.

$$2w \int_{-1.2}^{0.2} \sqrt{1.44 - y^2} dy$$

Answer: ______________________

6. Consider the function that is zero for $x > 10$ and $x < 10$ and whose graph for $0 \leq x \leq 10$ is given below.



(a) Explain why the function is a probability density function.

The area under the curve is equal to 1.

(b) $P(X < 4) =$ 0.2

(c) $P(4 < X < 9) =$ 0.75

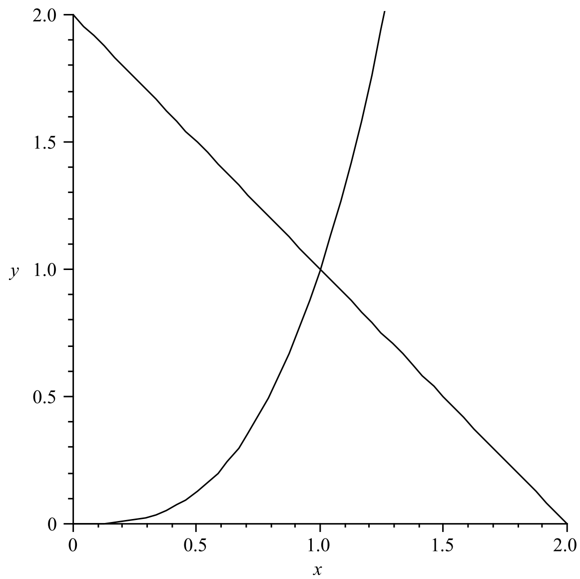
(d) mean = $2\sqrt{10}$

(e) median = 6

7. The curve $y = \sqrt{a^2 - x^2}$, $-\frac{3a}{4} \leq x \leq \frac{3a}{4}$ is rotated about the x-axis. Find the area of the resulting surface.

Answer: $3\pi a^2$

8. Find the centroid of the region bounded by the curves $y = x^3, x + y = 2$, and the y-axis.



Answer: (28/75, 92/105)

Area $= \int_0^1 (2 - x - x^3)dx = 5/4$

Moment about the y-axis $= \int_0^1 x(2 - x - x^3)dx = 7/15$

Moment about the x-axis $= \frac{1}{2}\int_0^1 \left((2 - x)^2 - (x^3)^2\right) dx = 23/21$

$\overline{x} = \frac{M_y}{A} = \frac{7/15}{5/4} = 28/75$

$\overline{y} = \frac{M_x}{A} = \frac{23/21}{5/4} = 92/105$