Math 113 Exam 3 Practice

February 22, 2010

Exam 2 will cover 7.7, 7.8, appendix G, 8.1-3 and 11.1-3. Please note that integration skills will still be needed for the material in 7.8 and chapter 8. For this exam you will **only** need to know the divergence test and the integral test. The other tests will be given in exam 3.

This sheet has three sections. The first section will remind you about techniques and formulas that you should know. The second gives a number of practice questions for you to work on. The third section give the answers of the questions in section 2.

Review

7.7 Approximate Integration

In this section we concentrate on two things: How can we efficiently approximate the solution to a definite integral, and how can we approximate the error.

You are expected to know how to calculate the following approximations of a definite integral $\int_a^b f(x)\,dx$:

(a) the left and right hand sum

Left Hand Sum: $L_f = \Delta x \sum_{k=1}^{n} f(x_{k-1})$
Right Hand Sum: $R_f = \Delta x \sum_{k=1}^{n} f(x_k)$

(b) the Trapezoid rule

$$T_f = \frac{\Delta x}{2}\left(f(a) + f(b) + 2\sum_{k=1}^{n-1} f(x_k)\right)$$

(c) the Midpoint rule

$$M_f = \Delta x \sum_{k=1}^{n} f\left(\frac{x_k + x_{k-1}}{2}\right)$$

(d) Simpson's rule

$$S_f = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n)\right].$$

Remember, n must be even to use Simpson's rule.

It is unlikely that you will need to calculate the left or right hand sums, but you will still need to know about them.

You are expected to know how each of these rules behave based on standard properties of the function (monotonicity, concavity, etc). For example, it is well known that the left hand sum is a lower bound and the right hand sum is an upper bound of the definite integral of increasing functions. What behavior is true for Midpoint and trapezoid?

You are expected to know the error estimates of Trapezoid, Midpoint and Simpsons:

Trapezoid

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

with $|f''(x)| \leq K$ on $[a, b]$.

Midpoint

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$

with $|f''(x)| \leq K$ on $[a, b]$.

Simpson's

$$|E_S| \le \frac{K(b-a)^5}{180n^4}$$

with $|f^{(4)}(x)| \le K$ on $[a, b]$.

You can be expected to use the error estimates in one of two ways: to estimate the error of the calculation for a particular value of n, or to find a value for n that gives an error no more than some stated value.

7.8 Improper Integrals

Remember, there are two types of improper integral:

- Infinite length: Integrals of the type
$$\int_{-\infty}^{a} f(x)\,dx, \quad \int_{a}^{\infty} f(x)\,dx.$$
- Unbounded integrand. Integrals of the type
$$\int_{a}^{b} f(x)\,dx, \quad \int_{c}^{a} f(x)\,dx$$
where f has an infinite discontinuity at a.

Remember, in each type, there is a "problem" that a definite integral cannot handle. We remove the problem by turning it into a limit:

$$\int_{a}^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_{a}^{b} f(x)\,dx$$

$$\int_{c}^{a} f(x)\,dx = \lim_{b\to a^-} \int_{c}^{b} f(x)\,dx.$$

Some things to remember when calculating improper integrals:

- Do not forget to set up an improper integral as a limit. You will likely have points deducted if you do not. It is the only way for the grader to tell that you know what you are doing.
- Watch out for infinite discontinuities in the middle of the interval. You must split the integral at the discontinuity in that case.

Appendix G

This section is somewhat of a departure from Chapter 8. This section is basically included to fill some gaps in your mathematical education. We can understand the natural logarithm better if we define it as an area integral, than if we just define it as the inverse of the natural exponential. In some ways, it is a more "natural" way to develop these two functions.

As you prepare for the exam, you should concentrate on two aspects:

- Use the area integral form of the natural logarithm to approximate it (by approximating the area).
- Be able to prove some of the properties of the natural logarithm using the area integral form of the natural logarithm.

8.1 Arc Length

To find the length of a curve given by $f(x), \quad a \le x \le b$, the formula is

$$s = \int_{a}^{b} \sqrt{1 + (f'(x))^2}\,dx.$$

Notice that there is a reason that this section is not in chapter 6. Arc length integrals can be difficult to solve. You may need any of the techniques from Chapter 7 to calculate the arc length.

If the integral is complicated, you may wish to simplify it first. for example, when f' is a fraction, you may wish to rewrite $1 + (f'(x))^2$ as a single fraction before you proceed.

8.2 Surface Area

Here are the important steps to keep in mind when solving this problem:

- First sketch the curve and identify the axis of rotation. Choose a variable, either x or y, to serve as our independent variable. If the curve is given as a function of x (e.g., $y = x^2 + 1$), then we will want to choose x as our independent variable, while if the curve is given as function of y (e.g., $x = y^3 - y + 1$), we will want to choose y as our independent variable. This is the variable with respect to which we will be integrating. If the curve is given implicitly (e.g., $x^2 + y^2 = 1$), then we may choose either variable and solve for the other variable.

- Next consider the circumferences of the circles formed by revolving points on the curve about the axis of rotation. The circumference of such a circle will be $2\pi r$, where r is the distance to the axis of rotation. This circumference should be expressed as a function of either x or y, whichever one we chose in the previous step to be our independent variable.

- The general formula for the area of a surface of revolution of this type is

$$S = \int_a^b (\text{circumference})\, ds$$

  where circumference is what we found above, and ds is either $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ or $\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$, according to whether x or y is our independent variable, respectively.

Examples:

Rotating $f(x)$, $a \le x \le b$ about the x axis: $\displaystyle\int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx$.

Rotating $f(x)$, $a \le x \le b$ about the y axis: $\displaystyle\int_a^b 2\pi x\sqrt{1 + (f'(x))^2}\, dx$.

Rotating $f(x)$, $a \le x \le b$ about the axis $y = c$: $\displaystyle\int_a^b 2\pi|f(x) - c|\sqrt{1 + (f'(x))^2}\, dx$.

Rotating $f(x)$, $a \le x \le b$ about the axis $x = d$: $\displaystyle\int_a^b 2\pi|x - d|\sqrt{1 + (f'(x))^2}\, dx$.

Exercise: Write the appropriate formulas for a function of y.

Note that you may need the same techniques to solve these integrals as you do in section 8.1.

8.3 Applications to Physics and Engineering

Fluid pressure

The calculation of fluid pressure on a plate vertically suspended in a liquid is

$$\int_a^b \omega D(h) L(h)\, dh$$

where $D(h)$ is the depth at h, and $L(h)$ is the width of the plate at h.

Centroid

There are three types of problems that you will need to be able to calculate.

- Using the property of sums of moments to be able to find centroids and center of mass. (Used in point masses, and in regions where the area is easily calculated.)

- Finding the moments with respect to the x and y axes, and the centroid of a region bounded by two functions. For example, if the region is bounded by

$$f(x) \le y \le g(x), \quad a \le x \le b,$$

  then we have:

$$\text{Moment about the y axis:} M_y = \int_a^b x(f(x) - g(x))\, dx$$

$$\text{Moment about the x axis:} M_x = \int_a^b \frac{1}{2}(f^2(x) - g^2(x))\,dx.$$

Since the area of the region is

$$A = \int_a^b (f(x) - g(x))\,dx,$$

the centroid is given by

$$\overline{x} = \frac{M_y}{A}, \quad \overline{y} = \frac{M_x}{A}.$$

- finding the center of mass of a plate. Here, we have a density. (mass is density times volume). If the density is constant, the moments are just the calculations above times the density. The mass of the object is also the area times the density. The center of mass, however, is the same as the calculation for the centroid. In the case of a density that is not constant, the situation is more complicated. However, since the homework only has constant density, that is all that need concern us for this exam.

You may need to rework the above equations for regions contained by functions of y.

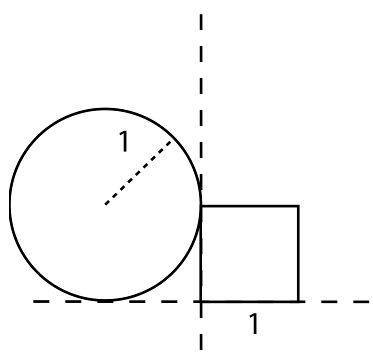
Geometric properties of the centroid

We now turn to some properties of the centroid which can simplify certain geometrical problems.

1. **Centroids of triangles are given geometrically** If you are trying to find the centroid of a triangle, you can do the following: Measure 2/3 along the line connecting a vertex to the midpoint of the opposite side. That's the centroid! (If you draw the lines from **each** vertex to the midpoint of the opposite side, they all intersect in the centroid.)

2. **Use symmetry** The centroid will always lie on a line of symmetry (if the object possesses one). For example, suppose we have a triangle with vertices at $(0,0)$, $(2,0)$ and $(1,5)$. This is an isosceles triangle. By the symmetry, it is clear that $\overline{x} = 1$. From the last item, since the line from the vertex above to the midpoint is vertical, we see that $\overline{y} = 5/3$.

3. **Moments Add!** If you are trying to find the moment of a complicated region, you can split the region into simpler regions and add the moments together. For example, suppose a circle of radius 1 and a square of length 1 are placed side by side. Where is the centroid of the system?



The centroid of the circle is $(-1,1)$. Thus, the moment of the circle about the y axis is -1 times the area of the circle: $-\pi$. Similarly, the moment of the circle about the x axis is π. Since the centroid of the square is at $(1/2, 1/2)$, the moments of the square about the y and x axes are both 1/2 (since the area of the square is 1). Thus, the total moment about the y axis is $-\pi + 1/2$. Also the total moment about the x axis is $\pi + 1/2$. Hence, the centroid is given by

$$\overline{x} = \frac{-\pi + 1/2}{\pi + 1}$$

and

$$\overline{y} = \frac{\pi + 1/2}{\pi + 1}.$$

4. **Other calculations look like moment calculations** Recall that when we developed the calculations for force due to fluid pressure, we took a small strip and multiplied the area by $\omega \times$ depth. That is like calculating a

moment! It should be no surprise, therefore, that if the force due to fluid pressure on the (submerged, vertical) plate is

$$\text{(weight density of liquid)} \times \text{(depth of centroid)} \times \text{(area of plate)}.$$

We also have the first theorem of Pappus for the calculation of volumes of rotation:

$$\text{Volume} = 2\pi \times \text{(distance from centroid to rotation axis)} \times \text{(area)}.$$

You should only use these formulas either when you already have the centroid, or when it is easy to find.

For example, suppose we wish to find the force on the side of a circular plate of radius 2 feet that is submerged in water to a point 5 feet from the top of the plate. Since the centroid of the plate is 7 feet from the surface, the force is given by

$$F = 62.5 \times 7 \times 4\pi = 437.5\pi lb.$$

Notice how much easier the calculation becomes! That is because the centroid was easy to find.

Sequences

Sequences are an important part of Chapter 11, because so much of what we do involves them. You need to be able to take a sequence and determine its behavior. Is is increasing or decreasing? Does it converge in the limit? What does it converge to? Some theorems may be of help here:

1. If a sequence converges, it is bounded.
2. If a sequence is bounded and is (eventually) increasing or decreasing, then it converges.
3. If a sequence $\{a_n\}$ matches a function f (i.e. $f(n) = a_n$) and

$$\lim_{n\to\infty} f(x) = L,$$

then the limit of the sequence is also L.

Rule 3 is useful because we can use everything we know about limits of functions to find limits of sequences. Since L'Hopital's rule is one of them, you should expect to use it. There are other rules about sums of sequences and products of sequences, etc. You are advised to review them in the text.

Sometimes we can determine whether a sequence is (eventually) increasing or decreasing by looking at a function $f(x)$ with $f(n)$ equalling the nth term of the sequence. If the function is increasing, the sequence is also.

Important Sequences

Some limits occur often enough that it is advisable to know about them in advance. For example, Dr. McKay expects all of his students to know the following:

1. If c is a real, positive number, then $c^{1/n} \to 1$.
2. If c is a real, positive number, then $\dfrac{1}{n^c} \to 0$.
3. $\dfrac{c^n}{n!} \to 0$.
4. $n^{1/n} \to 1$.
5. $\left(1 + \dfrac{c}{n}\right)^n \to e^c$.

All of the above limits except the third can be proven using L'Hopital's rule. The third is a bit tricky but can be done by noticing that the $(n+1)$st term of the series is $\dfrac{c}{n+1}$ times the nth term.

If you encounter these limits in a problem, you are welcome to use what you know about them and move on. (Unless the question *is* one of these limits.)

Recursive Sequences

Most sequences that we deal with have a rule we can apply to find the nth term of the sequence. **Recursive sequences**, however, only have a rule that allows us to find the nth term if we know all of the other terms that come before it. The most famous recursive sequence is the Fibbonaci sequence given by

$$a_1 = 1,\ a_2 = 1,\ a_n = a_{n-1} + a_{n-2}.$$

Generally, it is difficult to tell what a recursive sequence does. The theorems listed above can show that a recursive sequence converges. For example if the sequence can be shown to be bounded and is increasing, then it must have a limit. If a recursive sequence converges, then finding the limit of the sequence is easy: Let L represent the limit. Since the sequence converges to L, every sequence element in the recursion formula converges to L also. For example, suppose we wish to find the limit of

$$a_1 = 1, a_2 = 1, a_n = \frac{1}{2}a_{n-1} - \frac{1}{2^{n-1}}.$$

If we are reasonably sure the limit exists, we can replace a_n by L to get

$$L = \frac{1}{2}L - 0$$

You can then use this to solve for L. Make sure the series converges however, or you may arrive at the wrong conclusion. For example, suppose we have the recursive sequence

$$a_1 = 1, a_n = 2a_{n-1} - 1.$$

Using this technique gives $L = 2L - 1$, which yields $L = 1$. However, it is not hard to see that this sequence diverges.

Series

In this section we learned about convergent and divergent series. A series converges if the sequence of partial sums converge. There are some particular types of series that we learned about:

Geometric Series $\displaystyle\sum_{n=0}^{\infty} ar^n$. We learned that the geometric series converges to $\dfrac{a}{1-r}$ if $|r| < 1$ and diverges otherwise. We saw several applications where we could write a problem in terms of a geometric series.

Harmonic Series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. We saw by examination of the partial sums s_{2^n} that this diverges. The integral test also shows the divergence of this series.

Alternating Harmonic Series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ We know by a later test that the alternating Harmonic series converges. We know by the Maclaurin series of $\ln(1+x)$ that it converges to $\ln(2)$.

Telescoping Series This is a series where the partial sum collapses to the sum of a few terms. We can then take the limit of the partial sum to see what the series converges to.

Note that in 11.2 the **only** series whose sums we could calculate were geometric and telescoping.

Tests for Convergence-Divergence and Integral tests only

So far, we have learned about the following tests for convergence:

Divergence Test If $a_n \not\to 0$ then $\sum a_n$ diverges. This is an excellent test to start with because the limit is often easy to calculate. Keep in mind, however, if the limit **is** 0, then the Divergence test tells you nothing. You must try some other test.

p series If you recognize a series as a p series,

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

then you can use the fact that a p series converges when (and only when) $p > 1$.

Geometric series We discussed this in the last subsection.

Integral Test If we are trying to determine whether $\sum a_n$ converges, and there is a function $f(x)$ with $f(n) = a_n$, then the sum converges iff

$$\int_a^\infty f(x)\,dx.$$

(We assume that both the series $\{a_n\}$ and $f(x)$ are positive.) So the integral test is handy if the associated function can be integrated without too much difficulty.

Remember, the Integral test only works when the series has non-negative terms. If you have a series where the terms are all negative, then you can factor out the negative 1 and then use the test. If the series is alternating, you'll have to wait for 11.5.

Estimating the tail

In an infinite series, **the tail** is a term usually used to indicate the "last" part of the series. For example, if we wish to approximate the sum of the following convergent series,

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^3},$$

then we can write it as

$$\sum_{n=0}^{k} \frac{1}{(2n+1)^3} + \sum_{n=k+1}^{\infty} \frac{1}{(2n+1)^3}$$

The part that is still an infinite sum is called the tail. The sum of the tail is called the **error** of our approximation. If we can test convergence of a series by the integral test, then there is an easy way to find an estimate of the tail: Assume $f(x)$ is defined on $[b, \infty)$ for some b, and $f(n) = a_n$. Then

$$\int_{k+1}^{\infty} f(x)\,dx \le \sum_{n=k+1}^{\infty} \le \int_{k}^{\infty} f(x)\,dx.$$

For example suppose that we sum the first 5 terms of the above series:

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \approx \sum_{n=0}^{4} \frac{1}{(2n+1)^3}$$

$$= 1 + \frac{1}{27} + \frac{1}{125} + \frac{1}{343} + \frac{1}{729} = 1.049324231$$

How close is this? We find that

$$\int_5^\infty \frac{1}{(2x+1)^3}\,dx = \frac{1}{484} = 0.002066115702,$$

and

$$\int_4^\infty \frac{1}{(2x+1)^3}\,dx = \frac{1}{324} = 0.003086419753.$$

Thus, the error is between these two numbers.

Questions

Try to study the review notes and memorize any relevant equations **before** trying to work these equations. If you cannot solve a problem without the book or notes, you will not be able to solve that problem on the exam.

1. Estimate $\int_0^4 e^{-x^2}\,dx$ with midpoint, trapezoid, and simpson for n=4 and 8.

2. Calculate a bound on the error for the trapezoid calculation in the last question (n=4 only).

3. How large does n need to be in order for midpoint to have an error no larger than 10^{-5}?

4. How large does n need to be in order for Simpson's to have an error no larger than 10^{-5}?

For questions 5 to 11, evaluate the integral, or show that it diverges.

5. $\int_2^\infty \frac{1}{x\ln x}\,dx$

6. $\int_1^\infty \frac{\ln x}{x^4}\,dx$

7. $\int_1^\infty \frac{1}{(2x+1)^3}\,dx$

8. $\int_0^1 \ln x\,dx$

9. $\int_{\pi/2}^{\pi} \sec x\,dx$

10. $\int_1^\infty \ln x\,dx$

11. $\int_1^4 \frac{dx}{x^2-4}$

For questions 12 to 13, use the Comparison Theorem to show whether the improper integral converges or diverges.

12. $\int_1^\infty \frac{\tan^{-1} x}{x^2}\,dx$

13. $\int_1^\infty \frac{2+e^{-x}}{\sqrt[3]{x^2}}\,dx$

14. By comparing the areas, show that $2(\frac{1}{3}+\frac{1}{5}+...+\frac{1}{2n+1}) < \ln(2n+1) < 2(1+\frac{1}{3}+...+\frac{1}{2n-1})$.

15. Find the equation of the tangent line to the curve $y = 1/t$ that is parallel to the secant line AD, where $A = (1,1)$ and $D = (3,1/3)$.

16. What is $\log_{\sqrt{10}} 10 + \ln(\frac{1}{e}) + 2\log_5 \frac{5}{2} + \log_{25} 16$?

    (a) 5
    (b) $\frac{7}{2}$
    (c) 3
    (d) $\frac{3}{2}$
    (e) $3 - \log_5 2$
    (f) $5 - \log_5 2$

17. Which one is NOT right?

    (a) $\ln x$ means the area under the curve $y = 1/t$ from $t = 1$ to $t = x$;
    (b) $\lim_{x\to-\infty} e^x = \lim_{x\to-\infty} xe^x = 0$;
    (c) $\lim_{x\to0+} x\ln x = \lim_{x\to0+} \sqrt{x}\ln x = 0$;
    (d) $\int_1^\infty \frac{1}{x(\ln x)}dx$ is convergent;
    (e) $\frac{d}{dx}a^x = (\ln a)a^x$.

18. Find the arc length function for the curve $y = 2\sqrt{x}$ with initial point $(0,0)$.

19. Find the arc length function for the curve $y = 2x^{\frac{3}{2}}$ with initial point $(0,0)$.

20. Find the arc length of the curve $C : y = 3 + \frac{1}{2}\cosh(2x)$ for $0 \le x \le 1$.

21. Find the arc length for the curve $y = \sqrt{x-x^2} + \sin^{-1}(\sqrt{x})$ for $0 \le x \le 1$.

In questions (22) to (26), find the area of the surface formed by revolving the curve about the specified axis:

22. $y = 2x+1$, $1 \le x \le 3$, about the y-axis.

23. $x = \sin y$, $0 \le y \le \pi$, about the y-axis.

24. $y = 1+x^2$, $-1 \le x \le 1$, about the line $y = 2$.

25. $y = x^3+1$, $0 \le x \le 1$, about the line $y = 1$.

26. $\frac{x^2}{4} + \frac{y^2}{9} = 1$, about the x-axis.

27. A circular pipe of 4m diameter has a plate that regulates the flow. When the plate is in position, the flow is completely cut off. What is the force on the plate in this position, if the pipe is full of water on one side?

28. Find the force due to fluid pressure on one side of a plate in the shape of the upper half of a regular hexagon of diameter 8 feet if the hexagon is sitting vertically in water with the (flat) top of the hexagon at a depth of 15 feet.

29. A cylindrical barrel with height 2m and diameter 1m is filled to the brim with oil. If the weight density of oil is 900 kilograms per cubic meters, what is the force due to fluid pressure on the side of the barrel?

30. The masses m_i are located at the points P_i. Find the total moments M_y and M_x of the system, and find the center of mass of the system.
    $m_1 = 4, m_2 = 2, m_3 = 3$
    $P_1 = (1,2), P_2 = (-2,3), P_3 = (4,-1)$

31. Let R be the region bounded by $f(x) = \sin(x)$ and $g(x) = \cos(x)$ for $x \in [\pi/4, 5\pi/4]$. Sketch the region R, and find the centroid (center of mass) of R.

32. Let R be the region bounded by $f(x) = e^x$ and $g(x) = x^2$ for $x \in [0,1]$. Sketch the region R, and find the centroid (center of mass) of R.

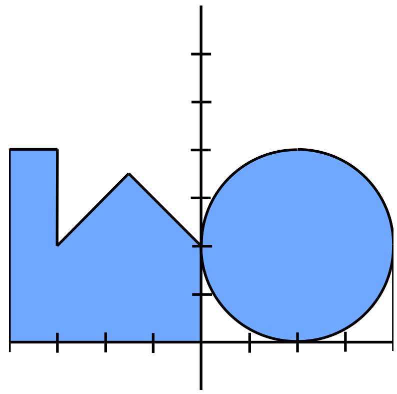
33. Let R be the region bounded by the curves $f(x) = 2-x^2$ and $g(x) = x^2$. Sketch the region R, and find the centroid (center of mass) of R.

34. Let R be the region consisting of two adjacent circular discs described by the equations $(x-1)^2+y^2 = 1$ and $(x-3)^2+y^2 = 1$. Use the theorem of Pappus to find the volume of the solid obtained by rotating R about the y-axis.

35. Let R be the cross shape formed by taking a square of side length 4, and cutting out of each corner a smaller square of side length 1. Use the theorem of Pappus to find the volume of the solid obtained by rotating R about one of its original sides.

36. Find the force due to fluid pressure on a vertical plate in the shape of an equilateral triangle (2 feet on a side), with the lower side parallel to the surface, and the upper vertex at a depth of 15 feet.

37. Find the centroid of the following object: (each tic mark is one unit)



Determine whether each sequence in 38 to 41 is convergent. State what it converges to, if applicable. Is the sequence increasing or decreasing? Is the sequence bounded?

38. $a_n = \dfrac{9^{n+1}}{10^n}$

39. $a_n = \cos(n\pi/2)$

40. $a_n = \dfrac{n \sin n}{n^2+1}$

41. $a_n = \dfrac{n^3}{1+n^2}$.

42. Find the value that the sequence given by $a_1 = 1$, $a_n = \frac{1}{a_{n-1}+1}$ converges to.

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

43. $\displaystyle\sum_{n=2}^{\infty} \frac{k^2}{k^2-1}$

44. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{k^2-1}$

45. $\displaystyle\sum_{n=0}^{\infty} \frac{3}{5^n}$

Determine whether each series in question 46 to 50 converges or diverges. State any convergence/divergence tests you use.

46. $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n}$

47. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\ln n}{n}\right)^2$

48. $\displaystyle\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+k^2}$

49. $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{(k^3+2)^2}$

50. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

Answers

1. $n = 4$**:** **Midpoint** .8861352469
   Trapezoid .8863185462
   Simpson .8362142646
   $n = 8$**:** **Midpoint** .8862269183
   Trapezoid .8862268966
   Simpson .8861963468
2. $|f''(x)| < 2$ on $[0, 4]$. $|E| < \frac{2}{3}$.
3. 731
4. $|f^{(4)}(x)| < 12$. $n = 52$.
5. Diverges.
6. $\frac{1}{9}$
7. $\frac{1}{36}$
8. -1
9. Diverges.
10. ∞
11. Diverges.
12. Converges. Make the comparison $\dfrac{\tan^{-1} x}{x^2} < \dfrac{\pi/2}{x^2}$.
13. Diverges. Make the comparison $\dfrac{2 + e^{-x}}{\sqrt[3]{x^2}} > \dfrac{2}{\sqrt[3]{x^2}}$.
14.
15. $y = -\frac{1}{3}x + \frac{2}{\sqrt{3}}$
16. c)
17. d)
18. $\sqrt{x + x^2} + \ln\left(\sqrt{x} + \sqrt{1 + x}\right)$
19. $L(x) = \dfrac{2(1 + 9x)^{\frac{3}{2}} - 2}{27}$
20. $\dfrac{1}{2} \sinh 2$
21. 2
22. $8\pi\sqrt{5}$
23. $2\pi(\sqrt{2} + \ln(\sqrt{2} + 1))$
24. $\frac{7}{8}\,\pi\,\sqrt{5} - \frac{17}{16}\,\pi\,\ln\left(-2 + \sqrt{5}\right)$
25. $\frac{\pi}{27}(10\sqrt{10} - 1)$
26. $\pi\left(18 + \frac{24}{\sqrt{5}} \ln\left(\frac{3+\sqrt{5}}{2}\right)\right)$
27. $8\pi\omega = 246300N$
28. $\omega(180\sqrt{3} + 40) = 21985.57$ lbs.
29. $2\pi\omega = 2\pi \cdot 9.8 \cdot 900 = 55417.69$ N
30. $M_y = 12, M_x = 11, (\bar{x}, \bar{y}) = (4/3, 11/9)$
31. $\left(\dfrac{3\pi}{4}, 0\right)$
32. $\left(\dfrac{9}{12e - 16}, \dfrac{3(5e^2 - 7)}{20(3e - 4)}\right)$
33. $(0, 1)$
34. $8\pi^2$
35. 48π
36. $62.5(15 + \frac{2}{3}\sqrt{3})\sqrt{3} = 1748.80lb$
37. $(\dfrac{-211 + 64\pi}{98 + 32\pi}, \dfrac{112 + 64\pi}{98 + 32\pi}) = (-0.05, 1.58)$
38. converges to 0, decreasing, bounded
39. diverges, not increasing or decreasing, bounded.
40. converges to 0, not increasing or decreasing, bounded.
41. diverges, increasing, bounded below.
42. $\dfrac{-1 + \sqrt{5}}{2}$
43. diverges by the Divergence Test
44. Converges to 3/2 (Telescoping sum)
45. Converges to 15/4 (geometric series)
46. Use the integral test
$$\int_1^\infty x^2 e^{-x} dx = \frac{5}{e}$$
Therefore it converges by the integral test
47. Use the integral test
$$\int_1^\infty \left(\frac{\ln x}{x}\right)^2 dx = 2$$
Therefore it converges by the integral test.
48. Use the integral test
$$\int_1^\infty \frac{\tan^{-1} x}{1 + x^2} dx = \frac{3\pi^2}{32}$$
Therefore it converges by the integral test
49.
$$\int_1^\infty \frac{x^2}{(x^3 + 2)^2} dx = \frac{1}{9}$$
Therefore it converges by the integral test.
50. We have to be careful here since the function is not defined at $k = 1$. By a change of variables, $k = n+1$ we see that $\displaystyle\sum_{k=2}^\infty \frac{1}{k \ln k} = \sum_{n=1}^\infty \frac{1}{(n + 1)\ln(n + 1)}$ and we can then use the integral test. The book notes that we can also simply change the limits of integration, though it does not state this as a theorem.
$$\int_1^\infty \frac{1}{(x + 1)\ln(x + 1)} dx = \infty - (-\infty)$$
Therefore it diverges by the integral test.