How to Read a Math Book

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Summary

Reading a math book is very different from reading other material. You may think your problems reading a math book are because you aren't smart enough or that you "aren't a math person," but that is wrong—it is because reading a math book is inherently hard, and doing it well takes lots of time and practice. Below are some basic steps that will help. It will still be hard, but you can do hard things; and the rewards are worth the effort.

How to Read a Math Book

1. *Plan to go very slowly.* Feeling frustrated that it is taking too long makes it hard to concentrate and learn.

Nota Bene: *Get unstuck by consulting other sources and asking other people.* At any point that you are confused, don't be afraid to find other sources* and ask other people. It is almost always a lot faster to talk to someone than to try to get unstuck by reading alone.†

2. *Identify the main ideas of the section.* Scan the section, read the introduction to the section, scan the section again. Try to answer the following questions before getting into details:
   (a) What are the main ideas?
   (b) Why are they being discussed?
   (c) How does this relate to what we already learned?
   (d) How does this relate to other things I know?
3. Now start your mid-depth reading: read all the definitions and theorems and propositions, etc., but not the proofs. Instead do the following.
   (a) With each new definition, ask yourself the key questions:
      i. What does this mean?
      ii. Why are they defining this?
      iii. What are some examples?
      iv. What are some unexamples?‡
      v. How does this relate to other things that I have learned?

* Despite what your previous teachers may have told you, *Wikipedia* is often a good source for mathematics.

† Mathematicians have the very bad habit of giving new, specialized definitions to regular, everyday words. For example, the word *map* to most people means something you use to find your way, but to a mathematician it means a *function*. If a sentence or a word you are reading doesn't make sense to you, it may be because the author is using a mathy meaning for a word like *map*. If there is no one nearby you can ask, try looking in the index, the glossary, or on *Wikipedia* to see if any of the words have a special math meaning.

‡ An *unexample* is an example that almost, but not quite, meets the conditions of the definition.

(b) With each theorem, corollary, proposition, or lemma, before reading the proof, ask essentially the same questions as you did with the definitions, but also ask:

i. What would happen if the various hypotheses were not met?

ii. Why does this deserve the name "theorem" (or not)?[§]

iii. Why is this is important enough to include in the book?

iv. What does this theorem tell me? How does it simplify or clarify the things we care about?

4. Now it is time to get serious. Do the proofs. With each proof:

(a) Skim first. Ask yourself:

i. What are the big ideas?

ii. How do they fit together?

iii. What parts look tricky? Why?

iv. How are the hypotheses used?

(b) Now read carefully, checking all the details. Continuously ask:

i. How do these details contribute to the big ideas?

ii. Why is this particular step necessary?

iii. How are the hypotheses being used?

iv. Can I do this step an easier way? Why or not?

v. How did the first person to prove this think of this step? What might have motivated her to try that?[¶]

vi. Have I seen another proof use similar techniques?

vii. Are there techniques I could use in another proof? One important reason to read proofs is to learn ideas and techniques that will help you prove your own, new results.

(c) *Sketch the main ideas of each proof.*

When you are done with the proof, again try to sketch the main ideas and key points of the proof without looking at the book.

5. *Outline the main ideas of the section.* When you are done with all the proofs, again try to outline the main ideas and key points of the section without looking at the book. Answer all the questions in Item 2 above.

6. *Do as many exercises as you can.* You may also want to go to the library and get another book on the same topic that has additional exercises.[||]

7. *Quiz yourself.* Quiz yourself on the main definitions, theorems, and ideas. This has a powerful effect on your ability to remember.

[§] Remember that the theorems are supposed to be the most important results in the section. A theorem with a name attached to it (e.g., *Cauchy's Theorem*, or *Spectral Mapping Theorem*) is almost always very important. Lemmas are usually just an intermediate result on the way to the proof of a big theorem, but occasionally a lemma has a name attached to it (e.g., *Gauss' Lemma*), and that is usually a sign that it really should have been called a theorem.

[¶] Mathematicians like to rewrite a proof in the slickest, shortest way possible. This hides all the ideas and mistakes that led to the final result and gives the false impression that only a genius could have thought of this proof.

[||] It is also important to get feedback on your answers. Try to find a professor or TA or friend who is willing to look over your work and help you recognize your errors. Some books also have some answers in the back, but only use them once you are finished with the problem—don't use them as a crutch.

8. *Explain it.* Explain it all to a classmate or friend. This helps you identify whether you really understand it, and it may also help the classmate. It also helps it all stick better.**

** If you have no friends who will listen to you explain, write up your own description of the material for someone else to read—for example on a blog, or by improving the appropriate Wikipedia page.