Name________________________________

Student Number______________________

Section Number______________________

Instructor__________________________

Math 112 – Fall 2006

Departmental Final Exam

Instructions:

- The time limit is 3 hours.
- Problem 1 consists of fill in the blank questions, each worth 1 point.
- Problems 2 through 8 are multiple choice questions, each worth 4 points.
  Their answers **MUST** be entered on the grid on page 2
- Work on scratch paper will not be graded. Do not show your work for problem 1 through 8.
- Write solutions to problems 9 through 18 on the exam paper in the space provided.
  Problem 9 is worth 10 points (2 points per part).
  Problems 10–18 are worth 6 points each. You must show your work to receive full credit.
- Please write neatly, and simplify your answers.
- Notes, books, and calculators are not allowed.
- Expressions such as ln(1), e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

For administrative use only:

1	/8
M.C.	/28
9	/10
10	/6
11	/6
12	/6

13	/6
14	/6
15	/6
16	/6
17	/6
18	/6
Total	/100

Math 112 – Winter 2006

Departmental Final Exam

Part I: Fill in the blank

1. *(8 points) Fill in the blanks with the correct answer. Include the constant of integration where appropriate.*

   (a) $f(x) = \dfrac{x}{\sin x}$ has a(n) ______________________________ discontinuity at $x = \pi$.

   (b) The fact that a continuous function on $[a, b]$ must have a maximum and minimum is called the ______________________________ theorem.

   (c) $\dfrac{d}{dx}(x^4 - 2x^2 + 6x - 5) =$ ______________________________

   (d) $\dfrac{d}{dx}(\tan^2 x) =$ ______________________________

   (e) $\displaystyle\int \frac{\sin x}{\cos x}\, dx =$ ______________________________

   (f) $\displaystyle\int \frac{1}{1 + x^2}\, dx =$ ______________________________

   (g) $\displaystyle\int e^{5x}\, dx =$ ______________________________

   (h) If $g(x)$ is an odd function, then $\displaystyle\int_{-3}^{3} g(x)\, dx =$ ______________________________

Part II: Multiple Choice

Problems 2 through 8 are multiple choice. Each multiple choice problem is worth 4 points. In the grid below fill in the square corresponding to each correct answer.

2	A	B	C	D	E	F	G	H	I	J
3	A	B	C	D	E	F	G	H	I	J
4	A	B	C	D	E	F	G	H	I	J
5	A	B	C	D	E	F	G	H	I	J
6	A	B	C	D	E	F	G	H	I	J
7	A	B	C	D	E	F	G	H	I	J
8	A	B	C	D	E	F	G	H	I	J

2. If $f(x) = \dfrac{1}{x-1}$, $f^{-1}(3) =$

(a) 0 (b) Undefined

(c) $\frac{1}{3}$ (d) $\frac{4}{3}$

(e) $\frac{2}{9}$ (f) $\frac{8}{9}$

3. $\displaystyle\lim_{x\to 5} \frac{|x-5|}{x-5} =$

(a) 1 (b) 5 (c) ∞ (d) -1 (e) -5 (f) Does not exist

4. On which of the following intervals is the function $f(x) = xe^{-x}$ concave up?

(a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) $(-\infty, 1)$ (d) $(1, \infty)$
(e) $(-\infty, 2)$ (f) $(2, \infty)$

5. Let $f(x)$ be a cubic polynomial with relative extreme points at $(1, 2)$ and $(3, 5)$. Then the value of $f(2)$ is

(a) 2 (b) 2.5 (c) 3 (d) 3.5
(e) 4 (f) 4.5 (g) 5 (h) 5.5

6. The function $f(x) = 3x^4 + 8x^3 - 18x^2 + 60$

a) has no local extreme points and no global extreme points.

b) has local minima at $x = -3$ and $x = 1$, a local maximum at $x = 0$, but no global extreme points.

c) has a local maximum at $x = 0$, a local minimum at $x = -3$, a global minimum at $x = 1$.

d) has a local maximum at $x = 0$, a global minimum at $x = -3$, a local minimum at $x = 1$.

e) has a local minimum at $x = 0$, a local maximum at $x = -3$, a global minimum at $x = 1$.

f) has a global maximum at $x = 0$, a local minimum at $x = -3$, a local maximum at $x = 1$.

7. Which of the following is a solution of $y' = 2xy$?

(a) $y = xy^2$ (b) $y = x^2y$ (c) $y = x^2$ (d) $y = e^{x^2}$
(e) $y = \ln x^2$ (f) $y = x$ (g) $y = 2x$

8. The mean value theorem for integrals states that for a continuous function f defined over $[a, b]$, there is a point $z \in [a, b]$ such that

(a) $f'(z) = \dfrac{f(b) - f(a)}{b - a}$

(b) $f(z) = \displaystyle\int_a^b f(x)\,dx$

(c) $f(z) = \dfrac{1}{b - a}\displaystyle\int_a^b f(x)\,dx$

(d) $f(z) = \displaystyle\int_a^b f'(x)\,dx$

(e) $f'(z) = \dfrac{1}{b - a}\displaystyle\int_a^b f(x)\,dx$

(f) $f(z) = \dfrac{1}{b - a}\displaystyle\int_a^b f'(x)\,dx$

(g) None of these

Part III: Written Solutions

For problems 9 - 18, write your answers in the space provided. Neatly show your work for full credit.

9. Calculate the following limits. If the limit is not finite, state if it is ∞ or $-\infty$, or, if neither cases apply, write does not exist.

   (a) $\lim_{x\to 0} \dfrac{\tan(3x)}{2x}$

   (b) $\lim_{x\to\infty} \dfrac{3x^2 - 2x + 4}{\sqrt{4x^4 - 3x^3}}$

   (c) $\lim_{x\to 3} \dfrac{x^2 - 3x}{x^2 - 9}$

   (d) $\lim_{x\to 2} \dfrac{x - 2}{\sqrt{x} - \sqrt{2}}$

   (e) $\lim_{x\to 0^+} \dfrac{1}{x \ln x}$

10. Differentiate the following:

    (a) $f(x) = x \sin(2x)$

    (b) $y = \dfrac{2x - 1}{x^3 + 3}$

    (c) $g(x) = \sqrt{e^{2x} - 1}$

11. Given the limit

$$\lim_{x \to 3}(3x - 2) = 7$$

and $\epsilon = \dfrac{1}{10}$, find an appropriate δ that satisfies the definition of the limit.

12. (a) Define the derivative of a function $f(x)$ as a limit.

(b) If $f(x) = 2x^2$, use the above definition to find $f'(3)$.

13. Find the equation of the line tangent to the curve $x^2 - 2y^2 = 3$ at the point $(2, 1/\sqrt{2})$.

14. Three competing species of antelope in the same grazing area have populations x, y, and z that satisfy the relationship

$$3x + 5y + 30z = 10000.$$

If x is increasing at the rate of 20 per year, and y is decreasing at the rate of 15 per year, how is z changing?.

15. In a particular apartment complex of 80 units, it is found that all units remain occupied when the rent is \$600 per month. For each \$20 increase in the rent, one unit becomes vacant, on average. Occupied units require \$80 per month for maintenance, while vacant units require none. Fixed costs for the complex are \$24,000 per month. What rent should be charged for maximum profit?

16. (a) Express the integral $\int_0^1 f(x)\,dx$ for a continuous function $f(x)$ as either a left-hand sum, right-hand sum or a Riemann sum over the subdivision $\left\{0, \frac{1}{n}, \frac{2}{n}, ..., 1\right\}$.

(b) Rewrite the following limit as a definite integral

$$\lim_{n\to\infty}\left(\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + ... + \cos\frac{2n\pi}{n}\right)\frac{2\pi}{n}$$

17. (a) Let $\int_a^b p(x)\,dx = 2$, $\int_a^b q(x)\,dx = 3$, $\int_a^b r(x)\,dx = 5$. Evaluate the integral

$$\int_a^b \left(3p(x) - 4q(x) + r(x)\right)\,dx + \int_c^c p(x)q(x)r(x)\,dx$$

(b) Let $f(x)$ be a continuous function with $f(0) = 2$, $f(1) = 5$. Evaluate the following:

$$\frac{d}{dx}\left(\int \sin(1 + x^2)\,dx + \int_0^1 \frac{\ln(1 + x^4)}{1 + x^4}\,dx\right) + \int_0^1 \frac{d}{dx}(2 + xf(x))\,dx$$

18. Evaluate the following integrals

(a) $\int \left(x\sqrt{x} + (3x^2)e^{x^3+2} \right) \, dx$

(b) $\int \frac{\cos 2x}{2 + \sin 2x} \, dx$

(c) $\int_1^e \frac{x^3 + 1}{x} \, dx$