Name:______________________________

Student ID:____________________________

Section:(See Bubble Sheet)________________

Instructor:_____________________________

Math 112 (Calculus I)
Final Exam Form A

April 19, 2013, 3:00-6:00 p.m.

Instructions:

- Work on scratch paper will not be graded.
- For questions 21 to 27, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer. Please write neatly.
- Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
- Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.
- Calculators are not allowed.

For Instructor use only.

#	Possible	Earned	#	Possible	Earned
MC	40		24b	4	
21	6		25	8	
22a	4		26a	4	
22b	4		26b	4	
22c	4		27a	4	
23	6		27b	4	
24a	4		27c	4	
Sub	68		Sub	32	
			Total	100	

Part I: Multiple Choice. Enter your answer on the scantron. Work will not be collected or reviewed.

1. Suppose A and B are two numbers such that $A + 2B = 20$. What is the maximum possible value of AB?

   a) 10 b) 20 c) 100 d) 25

   e) 42 f) 48 g) 50 h) 5

2. Water leaks out of a tank at the rate of $r(t) = 200 - 8t$ liters per minute, where $0 \leq t \leq 25$. Find the amount of water that leaks out in the first five minutes.

   a) 420 L b) -8 L c) 900 L d) 1100 L

   e) 860 L f) 2500 L g) 4500 L h) 160 L

3. Find the derivative $h'(x)$ of the function $h(x) = \dfrac{3e^x + 2x}{\sin x}$.

   a) $\dfrac{(3xe^{x-1} + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$ b) $\dfrac{(3e^x + 2x)\cos x}{\sin^2 x}$

   c) $\dfrac{3xe^{x-1} + 2}{\cos x}$ d) $\dfrac{(3e^x + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$

   e) $\dfrac{(3e^x + 2)\sin x}{\sin^2 x}$ f) $\dfrac{3e^x + 2}{\cos x}$

   g) $\dfrac{2}{\sin^2 x}$ h) None of these.

4. Use linear approximation to estimate $\sqrt{1.2}$.

   a) 0.9 b) 1.01 c) 1.5 d) 1.2

   e) 0.1 f) 0.98 g) 0.8 h) 1.1

5. If $\displaystyle\int_1^6 f(x)\,dx = 8$ and $\displaystyle\int_4^6 f(x)\,dx = 12$, find $\displaystyle\int_1^4 f(x)\,dx$.

   a) -2 b) 2 c) 20 d) -4

   e) 3 f) 4 g) -3

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6. Let $f(x) = 3x^5 + 5x^4 + 7$. On which of the following intervals is f increasing?

a) $(-1, 0)$
b) $(-4/3, 0)$
c) $(-\infty, -1)$ and $(0, \infty)$
d) $(-\infty, \infty)$
e) $(-1, \infty)$
f) $(-\infty, -4/3)$ and $(0, \infty)$
g) None of these.

7. $\dfrac{d}{dx}(\arccos(2x)) =$

a) $-\dfrac{2}{1+4x^2}$
b) $\dfrac{2}{\sqrt{1-4x^2}}$
c) $-\dfrac{2}{1-x^2}$
d) $-\dfrac{2}{\sqrt{1-4x^2}}$
e) $\dfrac{2}{1+x^2}$
f) $\dfrac{2}{1+4x^2}$
g) $\dfrac{1}{1+4x^2}$
h) $\dfrac{1}{\sqrt{1-4x^2}}$

8. Find the derivative $g'(x)$ of the function $g(x) = x^2 \cos x$.

a) $2x \sin x + x^2 \cos x$
b) $2x \sin x$
c) $\cos 2x$
d) $-2x^3 \sin x \cos x$
e) $2x \cos x - x^2 \sin x$
f) $-2x \sin x$
g) $-\sin 2x$
h) None of these.

9. If a function f is defined and twice differentiable on $(-\infty, \infty)$, $f'(2) = 0$, and $f''(2) = 4$, then

a) f has an inflection point at $x = 2$.
b) f is increasing in a neighborhood around $x = 2$.
c) f has a relative maximum at $x = 2$.
d) f has a relative minimum at $x = 2$.
e) f is decreasing in a neighborhood around $x = 2$.
f) We don't have enough information to prove that any of these are true.

10. What is the maximum value of $f(x) = 4x^2 - x^4 + 1$ on the interval $[-2, 2]$?

a) $y = 6$
b) $y = 0$
c) $y = 2$
d) $y = 4$
e) $y = 5$
f) $y = 1$
g) $y = 3$
h) $y = 9$
i) None of these.

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11. Let $h(x) = f(g(x))$, and let $g(2) = 1$, $g'(2) = 2$, $f(1) = 3$, $f'(1) = 5$, $f(2) = 3$, and $f'(2) = 7$. Find $h'(2)$.

a) 2 b) 35 c) 7

d) 14 e) 5 f) 10

g) 15 h) 21 i) 28

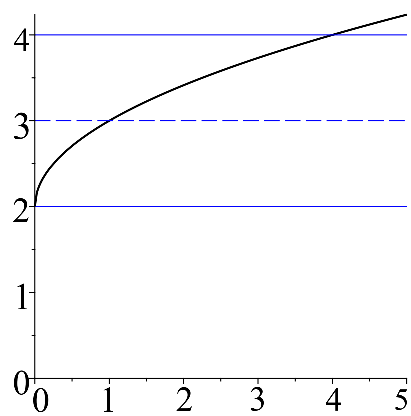
j) None of the above.

12. Find $\dfrac{dy}{dx}$ where $xy = \cos y$.

a) $-\dfrac{\sin y + y}{x}$ b) $-\dfrac{x \sin y + \cos y}{x^2}$ c) $-\dfrac{y}{(x + \sin y)}$

d) $-\sin y$ e) $\dfrac{\cos y}{x}$ f) None of the above.

13. Given the graph of the function $f(x) = \sqrt{x} + 2$, find the largest value for δ such that if $|x - 1| < \delta$, then $|(\sqrt{x} + 2) - 3| < 1$.



a) 1 b) 5 c) 4 d) 0.5

e) 2 f) 1.5 g) 0 h) 3

14. Suppose $y = 3x - 7$ is an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 1$. Find the values of $f(1)$ and $f'(1)$.

a) Cannot be determined without more information. b) $f(1) = 7$, $f'(1) = 3$

c) $f(1) = -7$, $f'(1) = 3$ d) $f(1) = 3$, $f'(1) = -4$

e) $f(1) = -1$, $f'(1) = 3$ f) $f(1) = -4$, $f'(1) = 3$

g) $f(1) = 3$, $f'(1) = -7$

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15. If for all x you know that $2x^2 + x - 2 \leq f(x) \leq 4x^4 + 2x^2 + x - 2$, do you have enough information to find $\lim_{x\to 0} f(x)$? If so, what is $\lim_{x\to 0} f(x)$?

a) Yes, -1

b) Yes, -2

c) Yes, 1

d) Yes, 0

e) Yes, 2

f) Yes, but none of the above numbers.

g) No, not enough information.

16. Evaluate $\displaystyle\int \frac{e^t}{(1-e^t)^2}\,dt$.

a) $\dfrac{1}{(1-e^t)} + C$

b) $e^t \ln(1-e^t)^2 + C$

c) $-\dfrac{1}{(1-e^t)^3} + C$

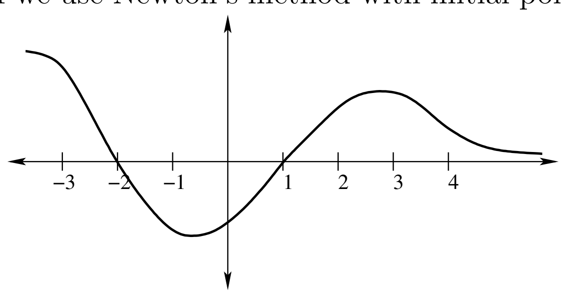
d) $\dfrac{e^t}{(1-e^t)} - \dfrac{2e^{2t}}{(1-e^t)^3} + C$

e) $-\dfrac{1}{(1-e^t)^2} + C$

f) $\dfrac{1}{(1-e^t)^3} + C$

g) $-\dfrac{1}{1-e^t} + C$

17. For the graph shown, if we use Newton's method with initial point $x_1 = 0$, what will happen?



a) We obtain a sequence of points converging to the root at $x = -2$.

b) We obtain a sequence of points diverging to ∞.

c) We obtain a sequence of points converging to the root at $x = 1$.

d) We obtain a sequence of points diverging to $-\infty$.

e) Newton's method will fail immediately.

f) None of the above.

18. Evaluate the sum $\displaystyle\sum_{n=0}^{99}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$.

a) -1 b) 101 c) $\dfrac{100}{101}$ d) 5050

e) $\dfrac{1}{101}$ f) 1 g) $-\dfrac{100}{101}$ h) $-\dfrac{1}{101}$

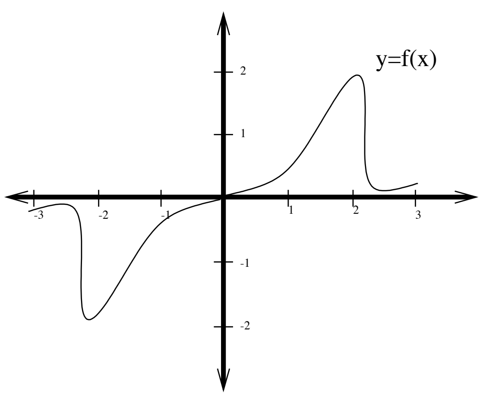
19. Find $\displaystyle\lim_{x\to -3^+}\frac{x}{x+3}$.

a) $-\infty$ b) -1 c) $\dfrac{1}{3}$ d) $-\dfrac{1}{2}$

e) 1 f) $-\dfrac{1}{3}$ g) ∞ h) $\dfrac{1}{2}$

i) 0

20. The following is the graph of a function $y = f(x)$. Which of the following most closely approximates the *definite integral* $\displaystyle\int_{-2}^{2} f(x)\,dx$?



a) 2 b) 6 c) 4 d) -6

e) -4 f) -2 g) 0

Free response: Write your answer in the space provided. Answers not placed in this space will be ignored.

21. (6 points) Use the definition of the derivative as a limit to find the derivative of $f(x)$ below. No credit will be given if a method besides the definition of the derivative is used.

$$f(x) = \frac{2}{x}.$$

22. Evaluate the limits.

(a) (4 points) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \frac{2}{n}$

(b) (4 points) $\displaystyle\lim_{x\to 1} \frac{\tan(x^2 - 2x + 1)}{(x-1)^2}$

(c) (4 points) $\displaystyle\lim_{x\to\infty} \frac{3x^2 - 2x + 1}{\sqrt{4x^4 - 3}}$

23. (6 points)

(a) State the hypotheses and conclusions of the Mean Value Theorem.

(b) Suppose f is a function that is continuous on $[0, \infty)$ and differentiable on $(0, \infty)$, and suppose that $f'(x) = 0$ for all $x \in [0, \infty)$. Use the Mean Value Theorem to show that $f(x)$ is a constant, equal to $f(0)$, for all x in $[0, \infty)$.

24. Differentiate the following.

(a) (4 points) $\dfrac{d}{dx} \displaystyle\int_0^{x^2} \frac{t^2}{\sin t + t^2}\, dt$

(b) (4 points) $\dfrac{d}{dx} \left(\sqrt{x}\right)^x$

25. (8 points) At 9:00 am, cyclist A is one mile west of cyclist B. Cyclist A is biking west at a rate of 4 mi/h, and cyclist B is biking north at a rate of 12 mi/h. How fast is the distance between them changing after 1 hour?

26. Integrate:

(a) (4 points) $\displaystyle\int_1^e \frac{t+4}{t}\,dt$

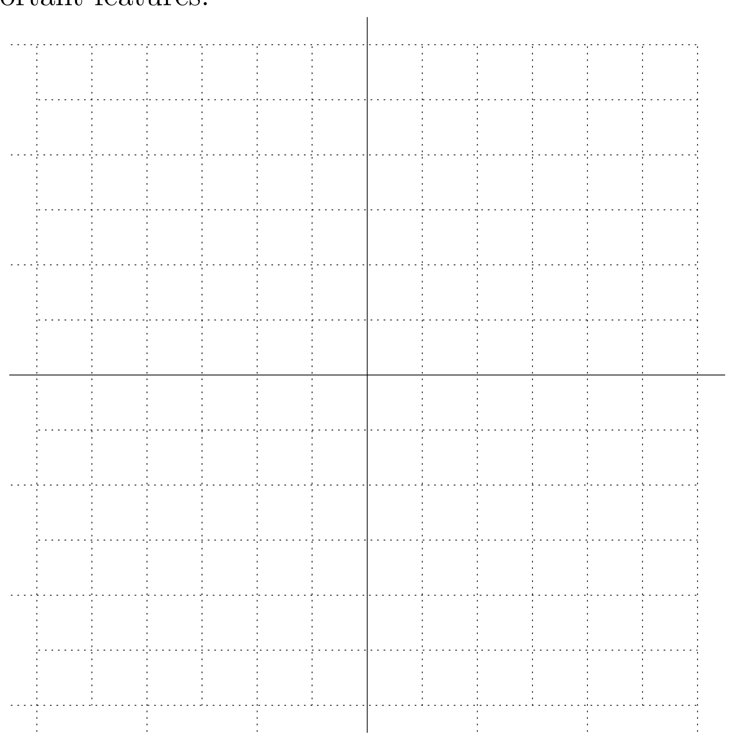
(b) (4 points) $\displaystyle\int \frac{\ln(x^3)}{x}\,dx$

27. In this problem, you will sketch the graph of $f(x) = x - \sin x$ on the domain $[-2\pi, 2\pi]$.

(a) (4 points) Find $f'(x)$, intervals on which f is increasing/decreasing, and local maxima/minima (if any) in the interval $[-2\pi, 2\pi]$.

(b) (4 points) Find $f''(x)$, intervals on which f is concave up/concave down, and inflection points (if any) in the interval $[-2\pi, 2\pi]$.

(c) (4 points) Sketch a graph of f on the interval $[-2\pi, 2\pi]$. Label all local extrema, points of inflection, and asymptotes (if any). Sketch the graph on a scale that makes it easy to see the important features.



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