Math 112 (Calculus I)
Final Exam Form A KEY

Part I: Multiple Choice. Enter your answer on the scantron. Work will not be collected or reviewed.

1. Suppose A and B are two numbers such that $A + 2B = 20$. What is the maximum possible value of AB?

   a) 10 b) 20 c) 100 d) 25

   e) 42 f) 48 g) 50 h) 5

   Solution: g)

2. Water leaks out of a tank at the rate of $r(t) = 200 - 8t$ liters per minute, where $0 \leq t \leq 25$. Find the amount of water that leaks out in the first five minutes.

   a) 420 L b) -8 L c) 900 L d) 1100 L

   e) 860 L f) 2500 L g) 4500 L h) 160 L

   Solution: c)

3. Find the derivative $h'(x)$ of the function $h(x) = \dfrac{3e^x + 2x}{\sin x}$.

   a) $\dfrac{(3xe^{x-1} + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$ b) $\dfrac{(3e^x + 2x)\cos x}{\sin^2 x}$

   c) $\dfrac{3xe^{x-1} + 2}{\cos x}$ d) $\dfrac{(3e^x + 2)\sin x - (3e^x + 2x)\cos x}{\sin^2 x}$

   e) $\dfrac{(3e^x + 2)\sin x}{\sin^2 x}$ f) $\dfrac{3e^x + 2}{\cos x}$

   g) $\dfrac{2}{\sin^2 x}$ h) None of these.

   Solution: d)

4. Use linear approximation to estimate $\sqrt{1.2}$.

   a) 0.9 b) 1.01 c) 1.5 d) 1.2

   e) 0.1 f) 0.98 g) 0.8 h) 1.1

   Solution: h)

5. If $\int_1^6 f(x)\,dx = 8$ and $\int_4^6 f(x)\,dx = 12$, find $\int_1^4 f(x)\,dx$.

a) -2 b) 2 c) 20 d) -4

e) 3 f) 4 g) -3

Solution: d)

6. Let $f(x) = 3x^5 + 5x^4 + 7$. On which of the following intervals is f increasing?

a) $(-1, 0)$ b) $(-4/3, 0)$ c) $(-\infty, -1)$ and $(0, \infty)$

d) $(-\infty, \infty)$ e) $(-1, \infty)$ f) $(-\infty, -4/3)$ and $(0, \infty)$

g) None of these.

Solution: f)

7. $\dfrac{d}{dx}(\arccos(2x)) =$

a) $-\dfrac{2}{1+4x^2}$ b) $\dfrac{2}{\sqrt{1-4x^2}}$ c) $-\dfrac{2}{1-x^2}$

d) $-\dfrac{2}{\sqrt{1-4x^2}}$ e) $\dfrac{2}{1+x^2}$ f) $\dfrac{2}{1+4x^2}$

g) $\dfrac{1}{1+4x^2}$ h) $\dfrac{1}{\sqrt{1-4x^2}}$

Solution: d)

8. Find the derivative $g'(x)$ of the function $g(x) = x^2 \cos x$.

a) $2x \sin x + x^2 \cos x$ b) $2x \sin x$ c) $\cos 2x$

d) $-2x^3 \sin x \cos x$ e) $2x \cos x - x^2 \sin x$ f) $-2x \sin x$

g) $-\sin 2x$ h) None of these.

Solution: e)

9. If a function f is defined and twice differentiable on $(-\infty, \infty)$, $f'(2) = 0$, and $f''(2) = 4$, then

a) f has an inflection point at $x = 2$.

b) f is increasing in a neighborhood around $x = 2$.

c) f has a relative maximum at $x = 2$.

d) f has a relative minimum at $x = 2$.

e) f is decreasing in a neighborhood around $x = 2$.

f) We don't have enough information to prove that any of these are true.

Solution: d)

10. What is the maximum value of $f(x) = 4x^2 - x^4 + 1$ on the interval $[-2, 2]$?

a) $y = 6$ b) $y = 0$ c) $y = 2$

d) $y = 4$ e) $y = 5$ f) $y = 1$

g) $y = 3$ h) $y = 9$ i) None of these.

Solution: e)

11. Let $h(x) = f(g(x))$, and let $g(2) = 1$, $g'(2) = 2$, $f(1) = 3$, $f'(1) = 5$, $f(2) = 3$, and $f'(2) = 7$. Find $h'(2)$.

a) 2 b) 35 c) 7

d) 14 e) 5 f) 10

g) 15 h) 21 i) 28

j) None of the above.

Solution: f)

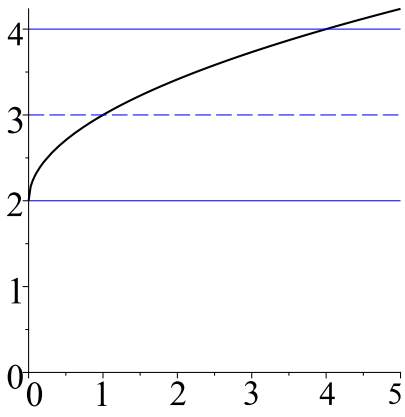
12. Find $\dfrac{dy}{dx}$ where $xy = \cos y$.

a) $-\dfrac{\sin y + y}{x}$ b) $-\dfrac{x \sin y + \cos y}{x^2}$ c) $-\dfrac{y}{(x + \sin y)}$

d) $-\sin y$ e) $\dfrac{\cos y}{x}$ f) None of the above.

Solution: c)

13. Given the graph of the function $f(x) = \sqrt{x} + 2$, find the largest value for δ such that if $|x - 1| < \delta$, then $|(\sqrt{x} + 2) - 3| < 1$.



a) 1 b) 5 c) 4 d) 0.5

e) 2 f) 1.5 g) 0 h) 3

Solution: a)

14. Suppose $y = 3x - 7$ is an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 1$. Find the values of $f(1)$ and $f'(1)$.

a) Cannot be determined without more information.

b) $f(1) = 7,\ f'(1) = 3$

c) $f(1) = -7,\ f'(1) = 3$

d) $f(1) = 3,\ f'(1) = -4$

e) $f(1) = -1,\ f'(1) = 3$

f) $f(1) = -4,\ f'(1) = 3$

g) $f(1) = 3,\ f'(1) = -7$

Solution: f)

15. If for all x you know that $2x^2 + x - 2 \leq f(x) \leq 4x^4 + 2x^2 + x - 2$, do you have enough information to find $\lim_{x \to 0} f(x)$? If so, what is $\lim_{x \to 0} f(x)$?

a) Yes, -1

b) Yes, -2

c) Yes, 1

d) Yes, 0

e) Yes, 2

f) Yes, but none of the above numbers.

g) No, not enough information.

Solution: b)

16. Evaluate $\displaystyle\int \frac{e^t}{(1 - e^t)^2}\, dt$.

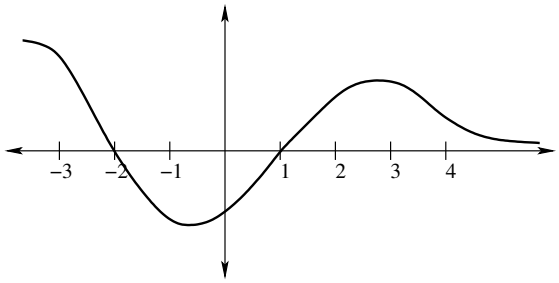
a) $\dfrac{1}{(1-e^t)}+C$ b) $e^t\ln(1-e^t)^2+C$ c) $-\dfrac{1}{(1-e^t)^3}+C$

d) $\dfrac{e^t}{(1-e^t)}-\dfrac{2e^{2t}}{(1-e^t)^3}+C$ e) $-\dfrac{1}{(1-e^t)^2}+C$ f) $\dfrac{1}{(1-e^t)^3}+C$

g) $-\dfrac{1}{1-e^t}+C$

Solution: a)

17. For the graph shown, if we use Newton's method with initial point $x_1 = 0$, what will happen?



a) We obtain a sequence of points converging to the root at $x = -2$.

b) We obtain a sequence of points diverging to ∞.

c) We obtain a sequence of points converging to the root at $x = 1$.

d) We obtain a sequence of points diverging to $-\infty$.

e) Newton's method will fail immediately.

f) None of the above.

Solution: c)

18. Evaluate the sum $\displaystyle\sum_{n=0}^{99}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$.

a) -1 b) 101 c) $\dfrac{100}{101}$ d) 5050

e) $\dfrac{1}{101}$ f) 1 g) $-\dfrac{100}{101}$ h) $-\dfrac{1}{101}$

Solution: c)

19. Find $\displaystyle\lim_{x\to -3^+}\frac{x}{x+3}$.

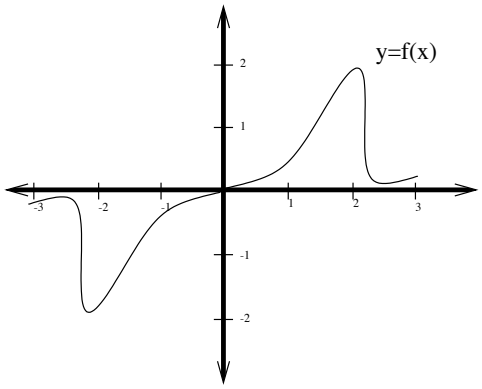
a) $-\infty$ b) -1 c) $\dfrac{1}{3}$ d) $-\dfrac{1}{2}$

e) 1 f) $-\dfrac{1}{3}$ g) ∞ h) $\dfrac{1}{2}$

i) 0

Solution: a)

20. The following is the graph of a function $y = f(x)$. Which of the following most closely approximates the *definite integral* $\int_{-2}^{2} f(x)\,dx$?



a) 2 b) 6 c) 4 d) -6

e) -4 f) -2 g) 0

Solution: g)

END OF EXAM