Name $\qquad$

Student Number $\qquad$

Section Number $\qquad$
Instructor $\qquad$

# Math 113 - Winter 2007 <br> Departmental Final Exam 

Instructions:

- The time limit is 3 hours.
- Problem 1 consists of 5 short answer questions.
- Problem 2 consists of $5 \mathrm{~T} / \mathrm{F}$ questions.
- Problems 3 through 7 are multiple choice questions.
- For problems 8 through 18 give the best answer and justify it with suitable reasons and/or relevant work.
- Work on scratch paper will not be graded. Do not show your work for problem 1 through 7.
- Please write neatly.
- Notes, books, and calculators are not allowed.
- Expressions such as $\ln (1), e^{0}, \sin (\pi / 2)$, etc. must be simplified for full credit.

For administrative use only:

| 1 | $/ 5$ |
| :---: | :---: |
| 2 | $/ 5$ |
| M.C. | $/ 15$ |
| 8 | $/ 7$ |
| 9 | $/ 7$ |
| 10 | $/ 6$ |
| 11 | $/ 7$ |


| 12 | $/ 6$ |
| :---: | :---: |
| 13 | $/ 9$ |
| 14 | $/ 7$ |
| 15 | $/ 7$ |
| 16 | $/ 6$ |
| 17 | $/ 7$ |
| 18 | $/ 6$ |
| Total | $/ 100$ |

# Math 113 - Winter 2007 <br> Departmental Final Exam 

## Part I: Short Answer and Multiple Choice Questions

Do not show your work for problems in this part.

1. Fill in the blanks with the correct answer.
(a) $1+x+x^{2}+\cdots$ is the Maclaurin series for
(b) $\int \ln (x) d x=$
(c) What technique must be used to integrate $\int \frac{x+1}{x^{3}+4 x} d x$ ?
(d) $\sum_{n=1}^{\infty}(-1)^{n-1} \frac{x^{2 n-1}}{(2 n-1)!}$ is the Maclaurin series for
(e) What substitution should be used to find $\int \sqrt{4-x^{2}} d x$ $\qquad$
2. True/False: Write T if statement always holds, F otherwise.

Let $\sum a_{n}=\sum_{n=1}^{\infty} a_{n}$ be an arbitrary series.
(a)_ $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ converges absolutely.
(b) $\quad \int \sec (x) d x=\ln |\sec (x)+\tan (x)|+C$
(c) The improper integral $\int_{1}^{\infty} \frac{d x}{x^{3}+1}$ converges.
(d) If $\sum_{n=1}^{\infty} a_{n}$ converges absolutely, then it converges.
(e) ___ If $\sum_{n=1}^{\infty} a_{n}^{2}$ converges, then $\sum_{n=1}^{\infty} a_{n}$ also converges.

Problems 3 through 7 are multiple choice. Each multiple choice problem is worth 3 points. In the grid below fill in the square corresponding to each correct answer.

3. Find the length of the graph of $y=x^{1 / 2}-\frac{x^{3 / 2}}{3}$ for $x \in[0,1]$.
(a) $\frac{4}{3} \sqrt{2}$
(b) $\frac{2}{3}$
(c) $\frac{8}{3}$
(d) $\frac{8}{9} \sqrt{3}$
(e) $\frac{4}{3}$
(f) None of the above.
4. Find the area between $y=x$ and $y=x^{2}$ for $x \in[-1,3]$.
(a) $\frac{29}{6}$
(b) $-\frac{9}{2}$
(c) $\frac{9}{2}$
(d) $\frac{17}{3}$
(e) $\frac{41}{3}$
(f) None of the above.
5. Let $A$ denote the region between the graph of $y=\cos x$ and the $x$ axis for $x \in\left[0, \frac{\pi}{2}\right]$. A solid is obtained by revolving $A$ about the line $x=-2$. Find the volume of this solid.
(a) $2 \pi\left(\frac{1}{2} \pi-3\right)$
(b) $2 \pi\left(\frac{1}{2} \pi+1\right)$
(c) $2 \pi\left(\frac{1}{2} \pi-1\right)$
(d) $4 \pi$
(e) $2 \pi\left(\frac{1}{2} \pi+2\right)$
(f) None of the above.
6. $\int_{0}^{2} \frac{x^{2}}{\sqrt{4-x^{2}}} d x=$
(a) $3 \pi / 2$
(b) $\pi / 3$
(c) $\pi / 2$
(d) $\pi$
(e) $2 \pi$
(f) None of the above.
7. Find the area enclosed by the spiral $r=f(\theta)=e^{\theta}$ where $\theta \in[0, \pi]$.
(a) $\frac{1}{4} e^{2 \pi}-\frac{1}{4}$
(b) $\frac{1}{2} e^{2 \pi}-\frac{1}{2}$
(c) $e^{\pi}-1$
(d) $\frac{1}{2} e^{\pi}-\frac{1}{2}$
(e) $\frac{1}{2} e^{2 \pi}-1$
(f) None of the above.

The answers to the multiple choice MUST be entered on the grid on the previous page. Otherwise, you will not receive credit.

## Part II: Written Solutions

For problems 8-18, write your answers in the space provided. Neatly show your work for full credit.
8. Determine whether each series converges absolutely, conditionally or fails to converge. State your conclusion next to the series.
(a) $\sum_{n=11}^{\infty} \frac{1}{(n-10) \ln n}$
(b) $\sum_{n=1}^{\infty} \frac{2 \sin (n)}{n^{2}+1}$
(c) $\sum_{n=0}^{\infty} \frac{n \cos (n \pi)}{n^{2}+1}$
9. Use a Maclaurin series to estimate

$$
\int_{0}^{0.06} \frac{\left(e^{x}-1\right)}{x} d x
$$

to four digit accuracy.
10. Give the Maclaurin series for

$$
G(x)=\frac{1}{\sqrt{x^{2}+1}}
$$

and determine the radius of convergence.
11. Find the power series of

$$
f(x)=\frac{1}{x+3}
$$

which is centered at 1 and give its interval of convergence.
12. Find $\int x^{2} e^{-x} d x$.
13. Evaluate the improper integral $\int_{1}^{\infty} \frac{d x}{x^{2}(x+1)}$ or show that it does not converge.
14. Use partial fraction decomposition to evaluate the following integral:

$$
\int \frac{6 x^{2}-3 x-18}{(x-5)\left(x^{2}+2 x+4\right)} d x
$$

15. (a) Find the sum of the series $S=\sum_{n=1}^{\infty} \frac{1}{2^{2 n-1}}$.
(b) Determine the convergence of the series $S=\sum_{n=1}^{\infty} \frac{1}{(2 n-1) 2^{2 n-1}}$.
16. Find the interval of convergence of the power series $S=\sum_{n=1}^{\infty} \frac{(x-3)^{n}}{n}$.
17. Find the power series of $\ln (1-x)$ centered at 0 and give its radius of convergence.
18. Consider the ellipse, $x^{2}-6 x+2 y^{2}+16 y+37=0$. Find the vertices, express in standard form, and sketch the graph.

