Samuel Sherman

118 test 1

Answer Key for Exam A

Complete the following exam and record your answers on the bubble sheet provided.

1. An experiment consists of rolling two dice and noting the result of each roll. Find the probability that the sum is not 7.

   (a) 5/6
   (b) 5/36
   (c) 1/4
   (d) 1/9
   (e) 29/36
   (f) None of the above

$\frac{E}{S} = \frac{36 - \#\text{ WAYS}}{6^2} = \frac{30}{36} = \boxed{\frac{5}{6}}$

(1,6)
(2,5)
(3,4)
(4,3)
(5,2)
(6,1)
} 6 WAYS TO GET SUM OF 7

2. If you flip a fair coin 3 times, what is the probability that you get at least two heads in a row? (Note: Either HHT or THH would be two heads in a row).

   (a) 1/8
   (b) 3/4
   (c) 3/8
   (d) 1/4
   (e) 1/2
   (f) None of the above

$\frac{3}{2^3} = \boxed{\frac{3}{8}}$

HHT
THH
HHH

3. MTV plans to visit 3 of the following 5 cities during spring break: Atlanta, Boston, Cincinnati, Dallas, and Eagle Mountain. An itinerary for their trip is a list of the three cities chosen in the order that they are visited. If the itinerary used by MTV is randomly selected from among all possible itineraries, what is the probability that Dallas is visited immediately before Eagle Mountain?

   (a) 3/10
   (b) 1/20
   (c) 1/10
   (d) 3/5
   (e) 3/20
   (f) None of the above

D E A
D E B
D E C
A D E
B D E
C D E
} 6 WAYS

$\frac{6}{60} = \boxed{\frac{1}{10}}$

5 CITIES, CHOOSE 3, ORDER MATTERS $= {}_5P_3 = 60$

4. An underclassman club is made up of 3 Freshman and 5 Sophomores. A committee of 3 is to be formed from the 8 so that at least one of each class is represented. Find the number of ways the committee could be formed.

(a) 45
(b) 63
(c) 70
(d) 90
(e) 120
(f) None of the above

$(3C_1)(5C_2) + (3C_2)(5C_1)$
$(3)(10) + (3)(5)$
$30 + 15$
45

5. What is the probability that 3 randomly selected people were born on different days of the week?

(a) 4/7
(b) 3/14
(c) 1/24
(d) 14/49
(e) 30/49
(f) None of the above

$\frac{E}{S} = \frac{7P_3}{7^3} = \frac{7 \cdot 6 \cdot 5}{7 \cdot 7 \cdot 7} = \frac{210}{343} = \frac{30}{49}$

6. How many unique words can be formed by rearranging the letters "bamboo"?

(a) 360
(b) 90
(c) 720
(d) 180
(e) 45
(f) None of the above

$\frac{6!}{2!1!1!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1 \cdot 1 \cdot 2 \cdot 1} = 180$

7. How many 4-letter codes can be formed using the alphabet $\{a, o, g, h, k\}$, if every code is required to have a vowel?

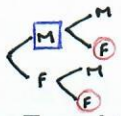
(a) $5^4 - 3^4$
(b) $4 \cdot P(5, 3)$
(c) $3^4 + 3^3 + 3^2$
(d) $\frac{5^4}{P(5,4)}$
(e) $P(4, 3) - P(4, 2)$
(f) None of the above

TOTAL - # w/out vowels
$5^4 - 3^4$

8. A doctor has 6 patients in the waiting room, 2 men and 4 women. Patients are called up in random order and seen in the order in which they were called. What is the probability that the first patient is a male given that the second is a female?

(a) 6/15
(b) 4/15
[(c)] 2/5
(d) 1/2
(e) 3/5
(f) None of the above

$$\frac{E}{S} = \frac{(2/6)(4/5)}{(2/6)(4/5)+(4/6)(3/5)} = \frac{(4/15)}{(4/15)+(2/3)} = \boxed{\frac{2}{5}}$$



9. There are 7 men and 3 women in a room. Two of these 10 people are selected at random. If both people are the same gender, then what is the probability that they are both women?

(a) 1/7
(b) 2/9
(c) 7/15
(d) 8/15
[(e)] 1/8
(f) None of the above

$$\frac{E}{S} = \frac{{}_3C_2}{{}_3C_2 + {}_7C_2} = \frac{3}{3+21} = \frac{3}{24} = \boxed{\frac{1}{8}}$$

10. If an urn contains 5 red balls and 5 green balls and 4 are drawn out at once, what's the probability that you have at least one of each color?

(a) $\frac{160}{C(10,4)}$
(b) $\frac{180}{C(10,4)}$
[(c)] $\frac{200}{C(10,4)}$
(d) $\frac{360}{C(10,4)}$
(e) $\frac{720}{C(10,4)}$
(f) None of the above

$$\frac{E}{S} = \frac{({}_5C_1)({}_5C_3) + ({}_5C_2)({}_5C_2) + ({}_5C_3)({}_5C_1)}{{}_{10}C_4} = \frac{50+100+50}{{}_{10}C_4} = \boxed{\frac{200}{{}_{10}C_4}}$$

11. A carton contains 4 red and 6 green apples. Two apples are drawn at random without replacement, and their color is noted. Determine the probability that at least one of the apples drawn is green.

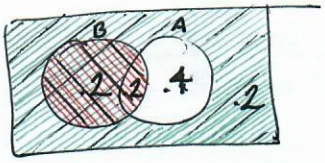
(a) 2/3
[(b)] 13/15
(c) 5/7
(d) 6/7
(e) 29/32
(f) None of the above

$$\frac{E}{S} = \frac{({}_4C_1)({}_6C_1) + ({}_4C_0)({}_6C_2)}{{}_{10}C_2}$$

$$\frac{24+15}{45} = \frac{39}{45} = \boxed{\frac{13}{15}}$$

12. Let S be a probability space with $A, B \subset S$. Suppose that $P(A) = 0.6$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$. Find $P(A' \cup B)$.

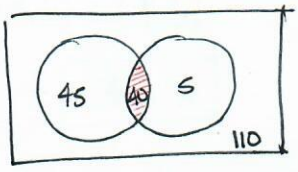
(a) 0.4
(b) 0.6
(c) 0.9
(d) 0.7
(e) 0.3
(f) None of the above



$.2 + .2 + .2 = \boxed{.6}$

13. Two hundred mathematicians attend a math conference. Forty five of these mathematicians have buck teeth. Eighty-five have skinny legs. One hundred and ten of these mathematicians don't have buck teeth and don't have skinny legs. How many have buck teeth and skinny legs?

(a) 110
(b) 90
(c) 55
(d) 50
(e) 40
(f) None of the above



$A + B - A \cap B = 90$

$45 + 85 - A \cap B = 90$

$130 - A \cap B = 90$

$\boxed{A \cap B = 40}$

14. A product code is to be formed with three distinct letters from the set $\{B, H, K, Q, T, V, Z\}$. How many different product code are there (note: BHK is a different product code than KBH).

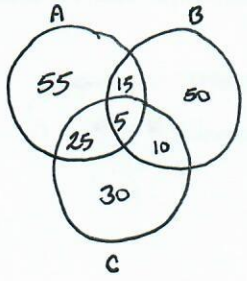
(a) 7!
(b) $\frac{7!}{3!}$
(c) 3!
(d) $\frac{7!}{4!}$
(e) $\frac{7!}{3!4!}$
(f) None of the above

$7P_3 = \boxed{\frac{7!}{4!}}$

ORDER MATTERS

15. Suppose A, B, C are subsets with: $n(A) = 100$, $n(B) = 80$, $n(C) = 70$, $n(A \cap B) = 20$, $n(B \cap C) = 15$, $n(A \cap B \cap C) = 5$, $n(A \cup B \cup C) = 190$. Find $n(A \cup B)$.

(a) 150
(b) 180
(c) 120
(d) 135
(e) 160
(f) None of the above



$A \cup B = A + B - A \cap B$

$= 100 + 80 - 20$

$= 160$

16. A presidency in a classroom consists of a president, vice-president, and secretary. Alice is one of 20 students in a class. If a presidency is chosen at random, what's the probability that she will be in a presidency?

(a) 3/20
(b) 1/20
(c) 1/19
(d) 3/10
(e) 1/18
(f) None of the above

$\frac{3 \text{ Vacancies}}{20 \text{ Students}} = \boxed{\frac{3}{20}}$

17. The universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ has subsets $A = \{1, 3, 4, 6\}$, $B = \{1, 2, 6, 7\}$, and $C = \{1, 4, 5, 6\}$. Find the set $(A \cap C) \cup (C \cap B')$.

(a) $\{1, 2, 3, 4, 5, 6, 8\}$
(b) $\{1, 2, 4, 5\}$
(c) $\{1, 4, 5\}$
(d) $\{1, 4, 5, 6\}$
(e) $\{4\}$
(f) None of the above

$(1,4,6) \cup (4,5) = \boxed{1,4,5,6}$

18. Three cards are selected at random from a standard deck of 52. What is the probability that exactly two cards are of the same suit? (Clarification: A standard deck is broken into four suits, namely hearts, spades, clubs, and diamonds, each with 13 cards).

(a) $156 \cdot \frac{C(13,2)}{C(52,3)}$
(b) $\frac{P(13,3) - P(13,2)}{P(52,3)}$
(c) $\frac{2,130}{255,000}$
(d) $(0.25)^3$
(e) $\frac{6084}{C(52,3)}$
(f) None of the above

$\frac{(4C_1)(13C_2)(39C_1)}{52C_3} = \frac{(4)(13C_2)(39)}{52C_3} = \boxed{\frac{156\,(13C_2)}{52C_3}}$

SUIT, 2 CARDS, 3rd CARD; TOTAL

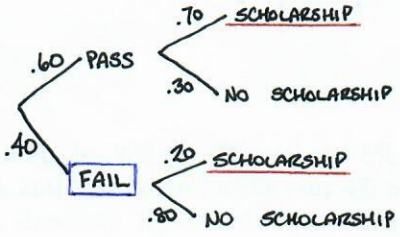
19. An experiment consists of rolling a fair die 4 times and noting the result of each roll. How many elements are there in the sample space for this experiment?

(a) 24
(b) 6^4
(c) 4^6
(d) $4 \cdot 36$
(e) $P(6, 4)$
(f) None of the above

4 ← # Trials
6 ← # Elements

20. Bob usually gets really good grades, but he's concerned about his PE class. Because of bad attendance and a pulled muscle, there's a 40% chance that he will fail the class. After finals the scholarship committee will review his file and decide whether to give him a scholarship. If Bob fails this class, there is only a 20% chance that he will get the scholarship. If he passes, then there is a 70% chance he will get the scholarship. Assuming he gets the scholarship, what is the probability that he failed the class?

(a) 0.25
(b) 0.08
[(c)] 0.16
(d) .19
(e) 0.20
(f) None of the above



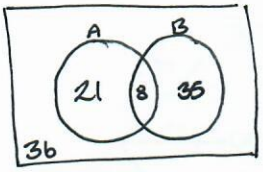
$\frac{E}{S} = \frac{(.40)(.20)}{(.40)(.20) + (.60)(.70)} = \frac{.08}{.5}$

$= \frac{16}{100}$

$= \boxed{.16}$

21. In a universe of 100 elements, suppose that $n(A') = 71$, $n(B) = 43$, and $n(A \cup B) = 64$. Find $n(A \cap B)$.

(a) 7
(b) 43
(c) 27
[(d)] 8
(e) 16
(f) None of the above



100 - 64 = 36

71 - 36 = 35

43 - 35 = 8

22. In Monopoly, you can go to jail by rolling doubles 3 times in a row. What is the probability that one can roll a pair of dice three times and get doubles all three times?

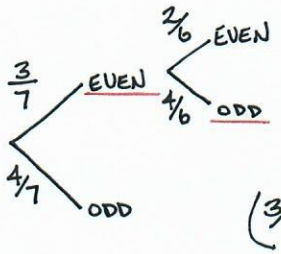
(a) $\left(\frac{1}{6}\right)^6$
(b) 1/7776
(c) 1/1296
[(d)] 1/216
(e) 1/36
(f) None of the above

$\frac{E}{S} = \frac{6}{36} = \frac{1}{6}$ = PROB OF ROLLING DOUBLES

$\left(\frac{1}{6}\right)^3 = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \boxed{\frac{1}{216}}$

23. Seven pool balls, numbered 1 through 7, lie on a pool table. One ball is drawn off at random and not replaced. Then another is drawn off at random. What is the probability that the first ball is even numbered and the second is odd numbered?

(a) 3/14
[(b)] 2/7
(c) 12/49
(d) 4/7
(e) 3/7
(f) None of the above



$\left(\frac{3}{7}\right) \cdot \left(\frac{4}{6}\right) = \frac{12}{42} = \boxed{\frac{2}{7}}$

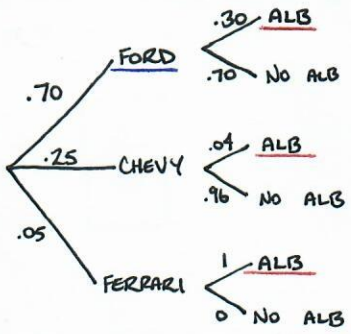
24. Two independent events A and B have probabilities $P(A) = 0.3$ and $P(B) = 0.7$. Find $P(A \cup B)$.

(a) 1
(b) 0.21
(c) 0.49
(d) 0.70
(e) 0.79
(f) None of the above

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= .3 + .7 - .21$$
$$= \boxed{.79}$$

25. In a large fleet of cars, 70% are Fords, 25% are Chevys, and 5% are Ferraris. Thirty percent of the Fords are equipped with anti-lock brakes, while only 4% of the Chevys have them. All Ferraris have anti-lock brakes, obviously. If you insist on a car with anti-lock brakes, and are given one at random, what is the probability that it will be a Ford?

(a) 7/8
(b) 7/9
(c) 7/11
(d) 7/12
(e) 7/13
(f) None of the above



$$\frac{E}{S} = \frac{(.7)(.3)}{(.7)(.3)+(.25)(.04)+(.05)(1)} = \frac{.21}{.21+.01+.05} = \frac{.21}{.27} = \boxed{\frac{7}{9}}$$