

Math 119

Pretest Review – Answers

Linear Equations

Find the slope of the line passing through the given points:

1. (2, -5); (0, 2)

Answer: $-7/2$

2. (3, -4); (3, 0)

Answer: Undefined

3. (-1, -2); (1, 4)

Answer: 3

4. Find the equation of the line with slope 2 that passes through (-2,-3). Write your answer in slope-intercept form.

Answer: $y = 2x + 1$

5. Find the general form of the equation of the line that passes through (-1,6) and is parallel to the graph of $2x + 3y - 4 = 0$.

Answer: $2x + 3y - 16 = 0$

6. Find the general form of the equation of the line that passes through (4,-2) and is parallel to the graph of $x - y - 3 = 0$.

Answer: $x - y - 6 = 0$

Quadratic Equations

Rewrite the following equations in general quadratic form:

7. $\frac{1}{x+1} - \frac{3}{x-2} = 5$

Answer: $5x^2 - 3x - 5 = 0$

8. $\frac{3x-2}{5} = x^2 + 1$

Answer: $5x^2 - 3x + 7 = 0$

9. $x + 1 = \frac{x}{x+2}$

Answer: $x^2 + 2x + 2 = 0$

Complete the square for the following functions:

10. $2x^2 - 8x + 5$

Answer: $2(x-2)^2 - 3$

11. $-x^2 - 4x - 7$

Answer: $-(x+2)^2 - 3$

12. $2x^2 - 4x$

Answer: $2(x-1)^2 - 2$

Factoring

Factor:

13. $4x^3y - 16x^2y - 28y$

Answer: $4y(x^3 - 4x^2 - 7)$

14. $x^3 + 2x^2 - 7x - 14$

Answer: $(x+2)(x^2 - 7)$

15. $5xy^2 + 5y^2 + 3ax + 3a$ (Group in pairs)

Answer: $(x+1)(5y^2 + 3a)$

16. $(x+3)^3 - 1$

Answer: $(x+2)(x^2 + 7x + 13)$

17. $a^3 - 64$

Answer: $(a-4)(a^2 + 4a + 16)$

18. $9y^2 - 64$

Answer: $(3y+8)(3y-8)$

Finding Zeros/Roots

19. Use Descartes's Rule of Signs to determine the possible number of positive and negative zeros:

$f(x) = x^3 + 1$

Answer: 0 positive, 1 negative

20. Given $f(x) = x^4 - 3x^3 + x^2 - 6x - 5$, determine the possible number of negative zeros.

Answer: Exactly One

21. List the possible rational zeros of the function:

$$f(x) = 3x^5 + 2x^2 - 3x + 2$$

$$\text{Answer: } \pm 2, \pm 1, \pm \frac{2}{3}, \pm \frac{1}{3}$$

22. List the possible rational zeros of the function:

$$f(x) = 4x^3 + 3x^2 - 5x + 6$$

$$\text{Answer: } \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$$

23. Use the fact that i is a zero of f to find the remaining zeros:

$$f(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$$

$$\text{Answer: } 2, 3, \pm i$$

24. Find all of the zeros of the function: $f(x) = x^4 + 25x^2 + 144$

$$\text{Answer: } \pm 3i, \pm 4i$$

25. Find all the real zeros of the polynomial function:

$$g(t) = t^3 + 3t^2 - 16t - 48$$

$$\text{Answer: } -4, -3, 4$$

26. Use the Intermediate Value Theorem to estimate the real zero in the interval $[1, 2]$:

$$-3x^4 + 2x^3 - x^2 + x + 2$$

Answer: Between 1.1 and 1.2

27. Use the Intermediate Value Theorem to estimate the real zero in the interval $[3, 4]$:

$$2x^3 - 5x^2 - 7x + 11$$

Answer: Between 3.0 and 3.1

28. Use the Intermediate Value Theorem to estimate the real zero in the interval $[0, 1]$:

$$3x^3 + 7x - 9$$

Answer: Between 0.9 and 1.0

Long and Synthetic Division

29. Find all of the real roots:

$$2x^3 + 5x^2 - x - 6 = 0$$

$$\text{Answer: } x = -2, -\frac{3}{2}, 1$$

30. Find all of the real zeros of the function:

$$f(x) = 2x^3 - 7x^2 + 7x - 2$$

$$\text{Answer: } x = \frac{1}{2}, 1, 2$$

31. Write as a product of linear factors:

$$f(x) = x^4 - 6x^3 - 4x^2 + 40x + 32$$

Answer:

$$(x - 4)(x + 2)(x - 2 + 2\sqrt{2})(x - 2 - 2\sqrt{2})$$

32. Simplify the rational expression:

$$\frac{2x^4 - x^3 - 4x^2 + x - 3}{2x + 3}$$

$$\text{Answer: } x^3 - 2x^2 + x - 1$$

33. Simplify the rational function:

$$f(x) = \frac{x^3 + 4x^2 - 3x + 10}{x + 5}$$

$$\text{Answer: } x^2 - x + 2$$

34. Simplify: $\frac{x^4 + 3x^3 - 3x^2 - 12x - 4}{x^2 + 3x + 1}$

$$\text{Answer: } x^2 - 4$$

Absolute Value

Solve the following equations for x :

35. $|x - 3| = 2$

$$\text{Answer: } x = \{1, 5\}$$

36. $|x| = 5$

$$\text{Answer: } x = \{-5, 5\}$$

37. $|x + 5| = -1$

Answer: No Solution

Simplify the following expressions involving absolute values:

$$38. \frac{|x^2 - 4x + 4|}{x - 2}$$

~~Answer: $|x - 2|$~~

Correction: $x - 2$

$$39. \left| \frac{x^2 - x - 6}{x - 3} \right|$$

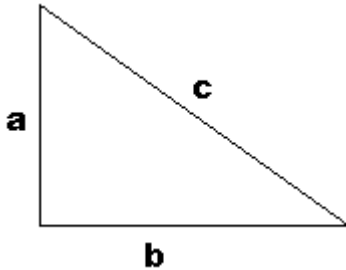
Answer: $|x + 2|$

$$40. \frac{|x^2 + 2x - 3| \cdot |x - 1|}{|x + 3|}$$

Answer: $(x - 1)^2$

Pythagorean Theorem

For questions 19-20, consider a right triangle with sides of length a , b , and c :



41. If the lengths of sides a and b are six and eight inches respectively, how long is side c ?

Answer: 10 in.

42. What is the length of side b in terms of the sides a and c ?

Answer: $\sqrt{c^2 - a^2}$

Distance Formula

Find the distance between the given points:

43. (2, 5); (-1, 9)

Answer: 5

44. (-1, 3); (5, 1)

Answer: $\sqrt{40}$

45. (10, -3); (3, 0)

Answer: $\sqrt{58}$

Simplifying Expressions

Simplify:

46. $\sqrt[4]{\frac{16x^4}{7y}}$ (Assume all variables represent positive real numbers)

Answer: $\frac{2x \cdot \sqrt[4]{343y^3}}{7y}$

47. $(\sqrt{x+2} + 2)^2$

Answer: $x + 6 + 4\sqrt{x+2}$

48. $\frac{x+1}{x-1} - \frac{3}{x+2}$

Answer: $\frac{x^2 + 5}{x^2 + x - 2}$

49. $\frac{27x^3y(x-2)}{3xy^2(x^2-4)}$

Answer: $\frac{9x^2}{y(x+2)}$

50. $\sqrt{x^3 - 3x + 2}$

Answer: $(x-1)\sqrt{x+2}$

51. $\frac{6\sqrt{x^3 - 3x + 2}}{\sqrt{3x^2 + 3x - 6}}$

Answer: $2\sqrt{3(x-1)}$

Rationalize the denominator and simplify:

$$52. \frac{1}{\sqrt{50}}$$

$$\text{Answer: } \frac{\sqrt{2}}{10}$$

$$53. \frac{2}{\sqrt{2}}$$

$$\text{Answer: } \sqrt{2}$$

$$54. \frac{12}{\sqrt{6}}$$

$$\text{Answer: } 2\sqrt{6}$$

Solving Equations

Solve for x or y:

$$55. (x^2 - 2x + 5)^{\frac{2}{3}} = 4$$

$$\text{Answer: } x = \{-1, 3\}$$

$$56. \frac{1}{x} - \frac{1}{x+1} = 1$$

$$\text{Answer: } x = \frac{-1 \pm \sqrt{5}}{2}$$

$$57. \frac{x}{x^2 - 9} + \frac{2}{x+3} = 3$$

$$\text{Answer: } x = \frac{1 \pm \sqrt{29}}{2}$$

$$58. \sqrt{2x+9} = x+5$$

$$\text{Answer: } x = -4$$

$$59. 3 + \sqrt{x-1} = x$$

$$\text{Answer: } x = 5$$

$$60. x - 1 = \sqrt{1 - 5x}$$

$$\text{Answer: No solution}$$

$$61. \frac{y}{y-3} + 3 = \frac{3}{y-3}$$

$$\text{Answer: No Solution}$$

$$62. \frac{y}{y-3} + 3 = \frac{-1}{y-3}$$

$$\text{Answer: } 2$$

$$63. \frac{y}{y-3} + 1 = \frac{-1}{y-3}$$

$$\text{Answer: } 1$$

$$64. x - 2 = \sqrt{x}$$

$$\text{Answer: } 4$$

$$65. 2\sqrt{x} = x$$

$$\text{Answer: } x = \{0, 4\}$$

$$66. 2\sqrt{x+3} = x+3$$

$$\text{Answer: } x = \{-3, 1\}$$

Inequalities

Solve each inequality:

Know both forms of the answer!

$$67. x > 3x - 2$$

$$\text{Answer: } x < 1$$

$$\text{or } x \in (-\infty, 1)$$

$$68. 3x < 4x + 1$$

$$\text{Answer: } x > -1$$

$$\text{or } x \in (-1, \infty)$$

$$69. x - 1 \leq 3x + 2$$

$$\text{Answer: } x \geq -3/2$$

$$\text{or } x \in [-3/2, \infty)$$

$$70. \frac{1-x}{x^2-x-6} > 0$$

$$\text{Answer: } x < -2, 1 < x < 3$$

$$\text{or } x \in (-\infty, -2) \cup (1, 3)$$

$$71. \frac{x+1}{x-3} \leq 0$$

$$\text{Answer: } -1 \leq x < 3 \\ \text{or } x \in [-1, 3)$$

$$72. \frac{x^2-1}{x} \geq 0$$

$$\text{Answer: } -1 \leq x < 0, x \geq 1 \\ \text{or } x \in [-1, 0) \cup [1, \infty)$$

$$73. x^2 + 1 > x + 3$$

$$\text{Answer: } x < -1, x > 2 \\ \text{or } x \in (-\infty, -1) \cup (2, \infty)$$

$$74. x^3 - 2x^2 + 6x + 3 \leq x^2 + 10x + 3$$

$$\text{Answer: } x \leq -1, 0 \leq x \leq 4 \\ \text{or } x \in (-\infty, -1] \cup [0, 4]$$

$$75. x^2 - 4x + 4 < x^3 - 6x^2 + 11x - 5$$

$$\text{Answer: } 1 < x < 3, x > 3 \\ \text{or } x \in (1, 3) \cup (3, \infty)$$

Solve each absolute value inequality:

Know both forms of the answer!

$$76. |x-3| < 2$$

$$\text{Answer: } 1 < x < 5 \\ \text{or } x \in (1, 5)$$

$$77. |x+5| \leq 0$$

$$\text{Answer: } x = -5$$

$$78. |x+2| \geq 3$$

$$\text{Answer: } x \leq -5, x \geq 1 \\ \text{or } x \in (-\infty, -5] \cup [1, \infty)$$

Logarithmic, Exponential and Logistic Functions

79. Write as the logarithm of a single quantity:

$$\frac{1}{2} [\ln(x+1) + 2 \ln(x-1)] + \frac{1}{3} \ln x$$

$$\text{Answer: } \ln(x^{1/3})(x+1)^{1/2}(x-1)$$

80. Write as the logarithm of a single quantity:

$$2 \ln|3x| + \ln|x+1| - 3 \ln|y|$$

$$\text{Answer: } \ln \left| \frac{9x^2(x+1)}{y^3} \right|$$

Simplify:

$$81. e^{2 \ln(x+1)}$$

$$\text{Answer: } (x+1)^2$$

$$82. \ln|5e^3|$$

$$\text{Answer: } 3 + \ln 5$$

$$83. \log_3 81$$

$$\text{Answer: } 4$$

$$84. \log_6 108$$

$$\text{Answer: } 2 + \log_6 3$$

85. The spread of a flu virus through a certain population is modeled by $y = \frac{1000}{1 + 990e^{-0.7t}}$

where y is the total number infected after t days. In how many days will 820 people be infected with the virus?

$$\text{Answer: } 12 \text{ days}$$

86. The amount remaining of a decaying sample is described by the function $m(t) = m_0 2^{-t/h}$ where m_0 is the initial amount, t is the number of hours, and h is the sample's half-life. Given an isotope of sodium with a half-life of 15 hours, find the amount remaining from a 2 g. sample after 20 hours.

$$\text{Answer: } 0.79 \text{ g}$$

87. The number of bacteria in a culture is given by the formula $n(t) = 1200e^{-0.35t}$

What is the half-life for this type of bacteria?

Answer: $t = 1.98$

Conic Sections

88. Find the standard equation of the parabola with vertex at $(0,0)$ and directrix $x = 7$

Answer: $y^2 = -28x$

89. Find the standard equation of the ellipse with center at $(0,0)$ a focus at $(2\sqrt{35},0)$ and minor axis of length 4.

Answer: $\frac{x^2}{144} + \frac{y^2}{4} = 1$

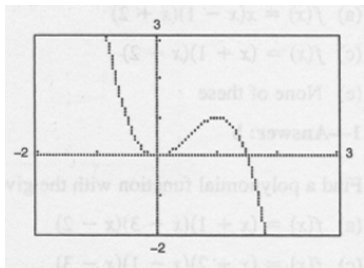
90. Find the foci of the hyperbola:

$$2y^2 - 9x^2 - 18 = 0$$

Answer: $(0, \pm\sqrt{11})$

Graphing and Translations

91. Determine the correct function for the given graph.



(a) $f(x) = 2x^3 - 3x^2$ (b) $f(x) = 3x^4 - 2x^3$

(c) $f(x) = 2x^3 + 3x^2$ (d) $f(x) = 3x^2 - 2x^3$

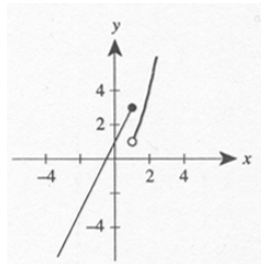
(e) None of these

Answer: (d)

92. Sketch the graph of the following

$$\text{function: } f(x) = \begin{cases} 2x + 1, & x \leq 1 \\ x^2, & x > 1 \end{cases}$$

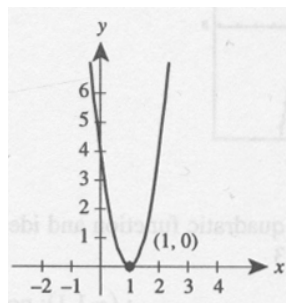
Answer:



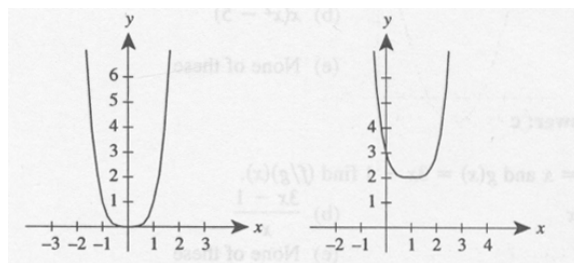
93. Graph the following function:

$$f(x) = 4x^2 - 8x + 4$$

Answer:



94. Use the graph of $y = x^4$ to find a formula for the function $y = f(x)$.

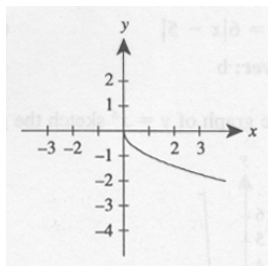


$y = x^4$

$y = f(x)$

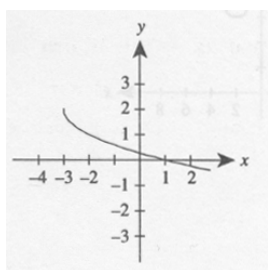
Answer: $y = (x - 1)^4 + 2$

95. Given the graph of $y = -\sqrt{x}$, graph $y = 2 - \sqrt{x+3}$.



$$y = -\sqrt{x}$$

Answer:



96. What sequence of transformations will yield the graph of $g(x) = (x+1)^2 + 10$ from the graph of $f(x) = x^2$?

Answer: Horizontal shift 1 unit to the left, vertical shift 10 units up

Domain

Determine the domain of the function:

97. $f(x) = \frac{3}{x-4}$.

Answer: $(-\infty, 4) \cup (4, \infty)$

98. $f(x) = \sqrt{6-x}$.

Answer: $(-\infty, 6]$

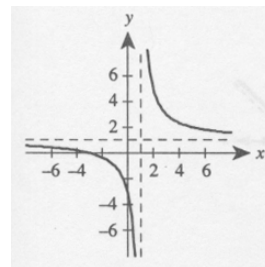
99. $f(x) = \begin{cases} \sqrt{x} & \text{if } x > 1 \\ |x| & \text{if } x < -1 \end{cases}$

Answer: $(-\infty, -1] \cup (1, \infty)$

Asymptotes

100. Graph the following function: $f(x) = \frac{3+x}{x-1}$

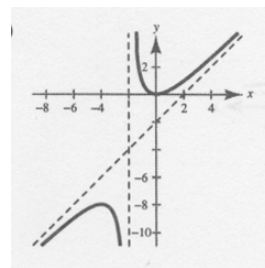
Answer:



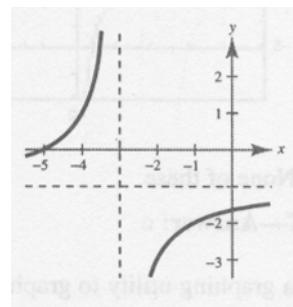
101. Graph the following rational function:

$$f(x) = \frac{x^2}{x+2}$$

Answer:



102. Match the graph with the correct function.



(a) $f(x) = \frac{x-5}{x+3}$ (b) $f(x) = \frac{5-x}{x+3}$

(c) $f(x) = \frac{-(x+5)}{x+3}$ (d) $f(x) = \frac{x+5}{x+3}$

(e) None of these

Answer: (c)

End Behavior

Describe the end behavior of the following functions:

103. $f(x) = \frac{x^2 - 4x - 5}{x - 3}$

Answer: The end behavior of $f(x)$ is that it follows the line $y = x - 1$. $f(x) \rightarrow \infty$ as $x \rightarrow \infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$.

104. $g(x) = \frac{x^3 - 2x^2 + 3}{x - 2}$

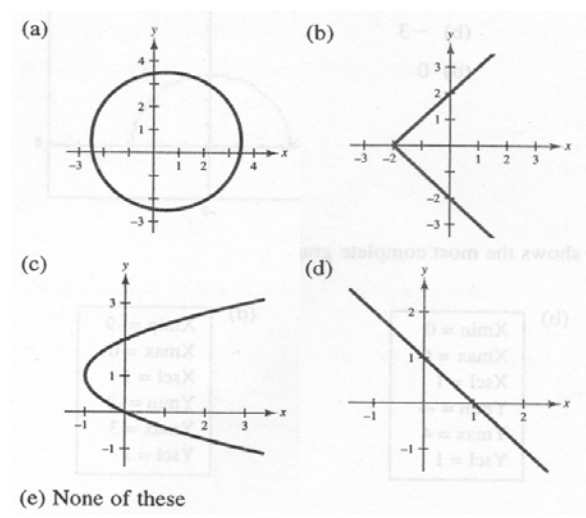
Answer: The end behavior of $g(x)$ is that it follows the parabola $y = x^2$; $g(x) \rightarrow \infty$ as $x \rightarrow \pm\infty$

105. $h(x) = \frac{x^2 - 2x + 3}{x^2 + 4x - 5}$

Answer: The end behavior of $h(x)$ is that it approaches the line $y = 1$; $h(x) \rightarrow 1$ as $x \rightarrow \pm\infty$

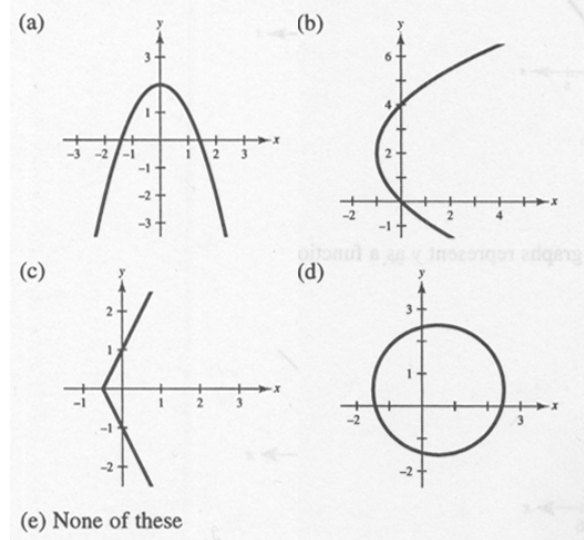
Vertical and Horizontal Line Tests

106. Use the vertical line test to determine in which case y is a function of x .



Answer: (d)

107. Which of the following graphs represent y as a function of x ?



Answer: (a)

One-to-One Functions

Determine whether each function is one-to-one. If it is, find its inverse.

108. $f(x) = \frac{7}{x + 2}$

Answer: yes, it is; $f^{-1}(x) = \frac{7 - 2x}{x}$

109. $y = \sqrt[3]{x^2 + 1}$

Answer: Not one-to-one

Inverse functions

Find the inverse of the following functions. State the domain and range of $f^{-1}(x)$.

110. $f(x) = -x^2 + 3$, for $x \geq 0$

Answer: $f^{-1}(x) = \sqrt{3 - x}$;

Domain: $(-\infty, 3]$; Range: $[0, \infty)$

111. $f(x) = \sqrt{2x}$, for $x \geq 0$

Answer: $f^{-1}(x) = \frac{x^2}{2}$; Domain: $[0, \infty)$;

Range: $[0, \infty)$

112. $f(x) = \sqrt{25 - x^2}$, for $0 \leq x \leq 5$

Answer: $f^{-1}(x) = \sqrt{25 - x^2}$;

Domain: $[0, 5]$; Range: $[0, 5]$

Composition of Functions

113. Evaluate $(f \circ g)(2)$ where $f(x) = 4x$ and $g(x) = -2x + 1$.

Answer: -12

114. Evaluate $(f \circ g)(3)$ where

$f(x) = -3x + 7$ and $g(x) = x^2 + 4$.

Answer: -32

115. Evaluate $(f \circ g)(3)$ where

$f(x) = \sqrt{x^2 + 7}$ and $g(x) = x^2 + 1$.

Answer: $\sqrt{107}$

Partial Fraction Decomposition

Find the partial fraction decomposition for the following expressions:

116. $\frac{7}{3x^2 + 5x - 2}$

Answer: $\frac{3}{3x - 1} - \frac{1}{x + 2}$

117. $\frac{5x + 3}{x^2 - 3x - 10}$

Answer: $\frac{4}{x - 5} + \frac{1}{x + 2}$

118. $\frac{-5x - 3}{x^2 - 9}$

Answer: $\frac{-3}{x - 3} - \frac{2}{x + 3}$

Sequences and Series

119. Write the first five terms of the geometric sequence with $a_1 = -3$ and $r = \frac{2}{3}$.

Answer: $-3, -2, -\frac{4}{3}, -\frac{8}{9}, -\frac{16}{27}$

120. Find the eighth term of the arithmetic sequence with $a_1 = 5$ and $d = 8$. (Assume that n begins with 1.)

Answer: 61

121. Determine whether the following is a geometric series, if so, find r : -2, 4, -8, 16, -32...

Answer: Yes, $r = -2$

Probability

122. The flags of seven different countries are to be displayed in a row. In how many different orders can they be flown?

Answer: 5040

123. Nine candidates are participating in an election. In how many different orders can they finish?

Answer: 362,880

124. There are eight football teams in the Mountain West Conference. In how many different orders can these eight teams finish in the final rankings?

Answer: 40320

125. Six friends are driving to a basketball game. In how many different orders can they arrive?

Answer: 720

126. A box holds 12 white, 5 red, and 6 black marbles. If 2 marbles are picked at random, without replacement, what is the probability that they will both be black?

Answer: $\frac{15}{253}$

Complex Numbers

Simplify the following complex numbers:

127. $i^2 + 3i + 2$

Answer: $1 + 3i$

128. $i^3 - 2i^2 + 4i - 3$

Answer: $-1 + 3i$

129. $i^4 + 4i^3 - 12i^2 - 13i + 1$

Answer: $14 - 17i$

Find the conjugate of the following complex numbers:

130. $4 - \sqrt{-3}$

Answer: $4 + \sqrt{3}i$

131. $\frac{3 + 4i}{16}$

Answer: $\frac{3}{16} - \frac{1}{4}i$

132. $3 - \sqrt{-1}$

Answer: $3 + i$