

Departmental Competency Exam  
MATH 112  
December 5, 1997

1. (15 pts) Determine whether each of the following limits exist. If it exists, compute it. If it does not exist, explain why.

(a)  $\lim_{x \rightarrow \pi} \frac{2x^2 - 6\pi x + 3\pi^2}{x^2 - \pi^2} =$

(b)  $\lim_{x \rightarrow 0} \frac{\tan x}{\sin 2x} =$

(c)  $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} =$

(d)  $\lim_{x \rightarrow \infty} \frac{\tan x}{x} =$

(e)  $\lim_{x \rightarrow 0} \frac{x - \frac{1}{2x}}{3x + \frac{4}{x}} =$

2. (9 pts) Give the  $\epsilon$ - $\delta$  definition for

$$\lim_{x \rightarrow a} f(x) = L.$$

Using this definition prove that

$$\lim_{x \rightarrow 3} (2x - 1) = 5.$$

3. (18 pts) Compute the derivatives of the following functions:

(a)  $(3x + 2)^{2/3}$

(b)  $\sin^2(\sin(\theta))$

(c)  $\log(x^5 + 1)$

(d)  $x \tan(x^2/2)$

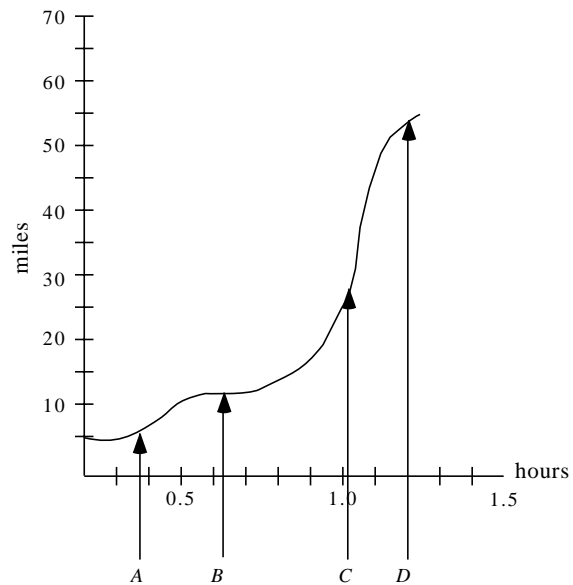
(e)  $\frac{\cos x}{x^2 + 2}$

(f)  $x^{x+1}$

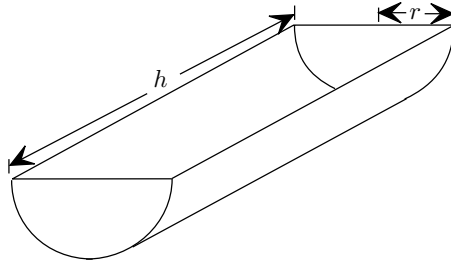
4. (4 pts) The illustration below shows the graph of the distance traveled by a car over time. Answer the following questions relative to this illustration.

(a) At which of the times  $A$ ,  $B$ ,  $C$ , or  $D$  is the car stopped.

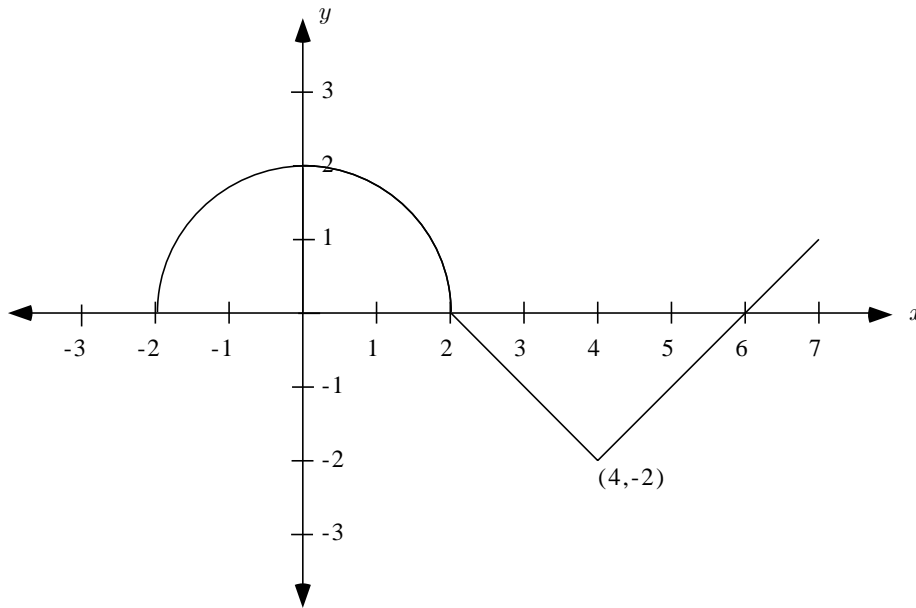
(b) At which of the times  $A$ ,  $B$ ,  $C$ , or  $D$  is the car traveling the fastest.



5. (8 pts) A metal trough with equal semicircular ends and open top is to have capacity  $64\pi$  cubic feet. Determine the radius  $r$  and the length  $h$  if the trough is to require the least material for its construction.



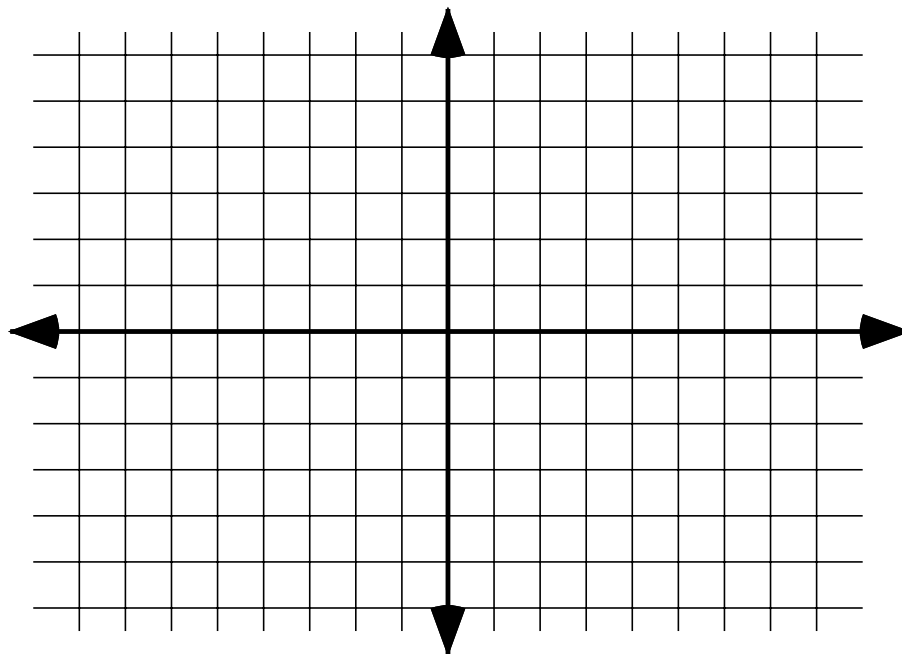
6. (11 pts) The graph of a function  $f$  consists of a semicircle and two line segments as shown below. Let  $g$  be the function defined by  $g(x) = \int_0^x f(t) dt$ .



- (a) Find  $g(4)$ .
- (b) Find all values of  $x$  on the open interval  $(-2, 7)$  at which  $g$  has a relative maximum. Justify your answer.
- (c) Write an equation for the line tangent to the graph of  $g$  at  $x = 4$ .

7. (5 pts) An investigation of a car accident showed that the driver applied his brakes and skidded 280 feet in 6 seconds. The speed limit was 30 miles per hour. Using the mean value theorem, determine if the driver was exceeding the speed limit at some point. (Hint: 30 miles per hour  $\doteq$  44 feet per second.)

8. (10 pts) Graph the function  $y = \frac{2x}{x^2 + 1}$ . Indicate maxima, minima, inflection points, and asymptotes, if they exist.



9. (4 pts) Given  $f'(1) = g(1) = 3$ ,  $f'(3) = g(3) = 2$ ,  $f(1) = g'(1) = 4$ , and  $f(3) = g'(3) = 7$ , find  $\frac{d}{dx}[f(g(x))]$  at  $x = 1$ .

10. (10 pts) Compute the following integrals:

(a)  $\int x^{3/2} dx =$

(b)  $\int x^{-3/2} dx =$

(c)  $\int_0^{\pi/2} \cos 2x dx =$

(d)  $\int e^{-3x} dx =$

(e)  $\int \sec^2 x dx =$

11. (6 pts) Using the definition of the definite integral as a limit of Riemann sums, show that

$$\int_0^1 x \, dx = \frac{1}{2}.$$