Math 113 Competency Exam

Fall 1997

In problems 1 through 4, evaluate the indefinite or definite integrals.

1. (5 pts) $\int (\ln x)^2 \, dx$

2. (5 pts) $\int \dfrac{x^2 + 3x + 3}{x(1+x)} \, dx$

3. (5 pts) $\displaystyle\int_0^{\sqrt{3}} \frac{dx}{(1+x^2)^2}$

4. (5 pts) $\displaystyle\int x\sqrt{1+x}\,dx$

5. Let $f(x) = \sqrt{4 - x^2}$.

(a) (8 pts) Calculate the area of the surface generated by revolving the graph of f about the x axis on the interval $[0, 1]$.

(b) (8 pts) Let S be the region in the x, y-plane that is bounded above by the curve $y = f(x)$, bounded below by the x axis, bounded on the left by the y axis, and bounded on the right by the line $x = 1$. Calculate the volume of the solid obtained by revolving S about the x axis.

6. (10 pts) Find the centroid (center of mass) of a plate of constant density in the x, y-plane that is bounded above by the curve $y = 1 - x^2$, bounded below by the line $y = -1$, bounded on the left by the line $x = -1$, and bounded on the right by the line $x = 1$.

7. (8 pts) Find the area enclosed by the (polar coordinate) curve

$$r = 1 - \cos\theta.$$

8. (5 pts) Calculate the improper integral $\int_{-1}^{1} \frac{dx}{x^2}$ or show that it is divergent.

9. (8 pts) Consider the following data:

x	$f(x)$
0	-3
1	-1
2	2
3	6
4	15

(a) Assuming $f(x)$ is a monotonic (increasing) function, find the best possible upper and lower bounds for

$$\int_0^4 f(x)\,dx.$$

(b) Approximate the integral using all the data and Simpson's rule.

10. (10 pts) Suppose x is positive but very small. Arrange the following expressions in increasing order:

$$x, \quad \sin x, \quad x\cos x, \quad e^x - 1, \quad xe^x.$$

(Hint: You may want to compare Taylor series for these functions.)

11. (8 pts) Suppose f is a function that satisfies $f(2) = 1$, $f'(2) = 2$, and $f''(2) = 3$.

(a) Use a second degree Taylor polynomial to approximate $f(2.1)$.

(b) If $f'''(x)$ exists and $|f'''(x)| \leq 4$ for every x, find (with justification) an upper bound for the difference between $f(2.1)$ and the approximation of $f(2.1)$ you gave in part (a).

12. (9 pts) For which numbers p in $(-\infty, \infty)$ do the following series converge?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{p^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^p$

13. (6 pts) Suppose

$$f(x, y) = [g(x)]^{h(y)}.$$

Express the following in terms of $g(x)$, $h(y)$, $g'(x)$, and $h'(y)$:

(a) $\dfrac{\partial}{\partial x} f(x, y)$

(b) $\dfrac{\partial}{\partial y} f(x, y)$