

MATH 113
Final Exam
Winter 1997

35 pts 1. Evaluate each of the following integrals. Show all essential steps:

a) $\int x^2 \sin x dx$

b) $\int \cos x(1 + \sin x)^3 dx$

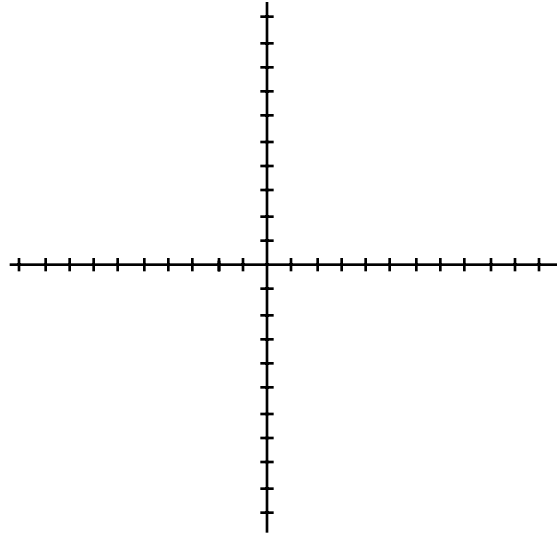
c) $\int \frac{t^2}{\sqrt{9-t^2}} dt$

d) $\int_0^1 \frac{dx}{(x+2)(x+1)}$

e) $\int_0^{\infty} \frac{1}{1+x^2} dx$

- 2 pts 2. a) Sketch the region bounded by the lines described by the following equations

$$y = 2^x; \quad y = 0; \quad x = 1; \quad x = 3$$



- 5 pts b) Set up the integral that describes the volume of the solid created by rotating the region around the y -axis. (DO NOT EVALUATE THE INTEGRAL.)

Volume = _____

- 4 pts 3. Set up the definite integral giving the arc length of

$$y = \frac{1}{2}x^2 + 4, \quad 0 \leq x \leq 4.$$

DO NOT INTEGRATE.

arc length = _____

- 2 pts 4. a) Of the numerical integration methods: Simpson's Rule, Left-hand Riemann sum, and Trapezoidal Rule, which is the most accurate and which is the least accurate in general?

Most accurate _____ Least accurate _____

- 5 pts b) Use Simpson's Rule to approximate $\int_0^4 \sqrt{x} dx$ using the points $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4$. Leave answer in terms of square roots.

5. For each of the following series, say whether or not it converges and give a reason for your answer.

4 pts

a)
$$\sum_{n=1}^{\infty} \frac{n^3}{5n^6 + 2}$$

4 pts

b)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

4 pts

c)
$$\sum_{k=1}^{\infty} \frac{\ln k}{k}$$

4 pts

6. a) What is the Taylor series of a function $f(x)$ about $x = a$?

2 pts

- b) Give the Taylor series about $x = 0$ of $f(x) = e^x$.

4 pts

- c) Give the Taylor series about $x = 0$ of $f(x) = \frac{1}{1+x^2}$

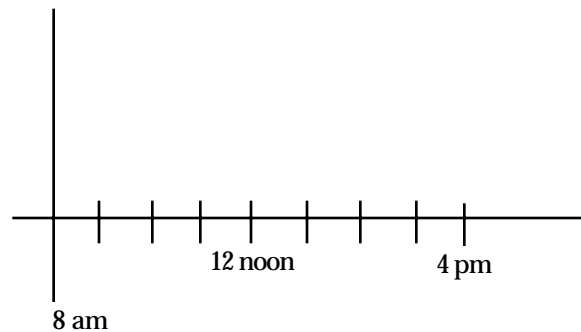
10 pts 7. Determine the interval of convergence of the power series

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(x+1)^k}{k}.$$

8. The ticket office opens at 8 am to sell student basketball tickets. At that time, there are already 600 students in line and students are arriving at the rate of 500 students per hour. The rate at which students arrive increases steadily (linearly) until 1 pm, when the rate is 1500 students per hour. The rate at which students arrive then decreases steadily to 0 at 4 pm, when the ticket office closes. The ticket office can serve students at the rate of 1000 students per hour.

2 pts

a) Sketch a graph of the rate at which students arrive at the ticket office as a function of time.



5 pts

b) Let t be the number of hours after 8 am. Find a formula for the length of the line l as a function of t , and use it to find the time when the line disappears.

$$l(t) = \underline{\hspace{2cm}}$$

Line disappears when time is am/pm

2 pts

c) At what time does the line begin to reappear?

Line reappears when time is _____ am/pm

6 pts

d) Let T be the number of hours after the time the line begins to reappear. Find a formula for the length of the line L as a function of T (remember the rate changes at 1 pm) and use it to find when the line is longest. How many students are in it then?

$L(T) =$ _____

Line is longest when time is _____ am/pm

Number of students then is _____

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a) $\int \sqrt{9-t^2} dt$

b) $\int_0^{\infty} \frac{dx}{4+x^2}$

c) $\int_0^1 \frac{dx}{(x+3)(x+2)}$

d) $\int x^2 e^{2x} dx$

e) $\int x^2 (2+x^3)^4 dx$

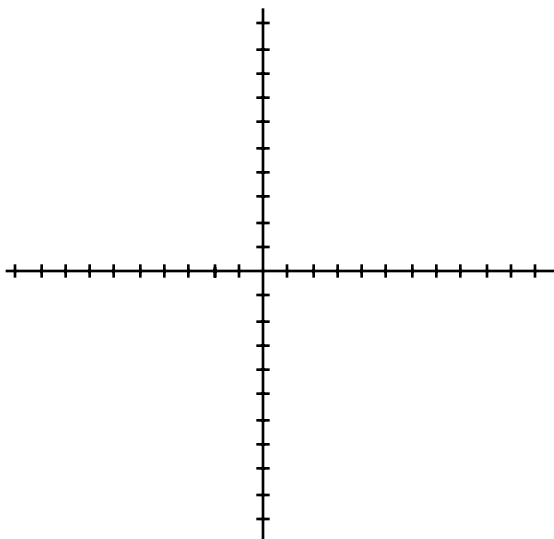
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- 5 pts b) Use Simpson's Rule to approximate $\int_0^4 \sqrt{x} dx$ using the points $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4$. Leave answer in terms of square roots.

- 2 pts 3. a) Sketch the region bounded by the lines described by the following equations

$$y = x^2 - 1; \quad y = 0; \quad x = 1; \quad x = 3$$



- 5 pts b) Set up the integral that describes the volume of the solid created by rotating the region around the y-axis. (DO NOT EVALUATE THE INTEGRAL.)

Volume = _____

- 4 pts 4. Set up the definite integral giving the arc length of

$$y = (x - 1)^2, \quad 0 \leq x \leq 4.$$

DO NOT INTEGRATE.

- 4 pts 5. a) What is the Taylor series of a function $g(x)$ about $x = a$? arc length = _____

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b) Give the Taylor series about $x = 0$ of $g(x) = \sin x$.

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c) Give the Taylor series about $x = 0$ of $g(x) = \frac{1}{1-x^2}$

6. For each of the following series, say whether or not it converges and give a reason for your answer.

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a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

4 pts

b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3 + k}}$

4 pts

c) $\sum_{n=1}^{\infty} \frac{n^3}{5n^4 - 2}$

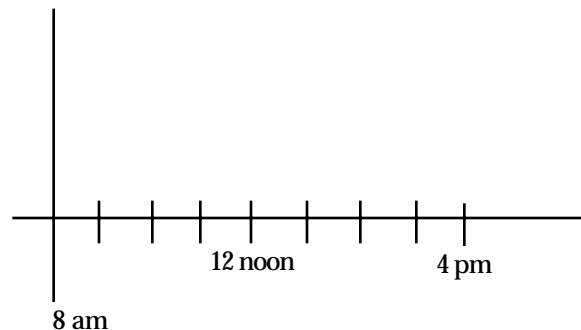
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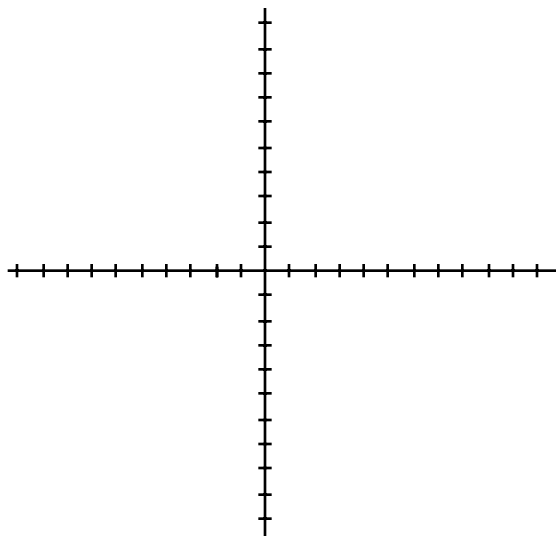
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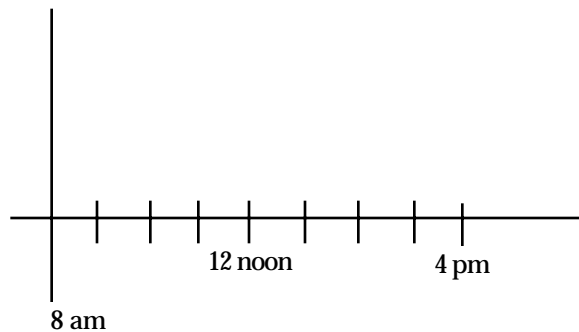
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