MATH 113 – WINTER 2000

Departmental Final Exam

Problems 1 through 8 are multiple choice. Select the best answer and fill in the corresponding bubble. Please make certain that your name and social security number are coded on the bubble sheet.

3 pts ea. 1. The series $\sum_{n=1}^{\infty} 1/n^p$ converges if and only if

a) $p < 1$ b) $p \le 1$ c) $p \ge 1$

d) $p > 1$ e) $p \le 2$ f) $p > 2$

g) $p > 0$ h) $p < 2$ i) none of these

3 pts 2. If $f(x, y) = (x^2 + x^3y + yx)(x + 2y)$, then $\frac{\partial f}{\partial x}(1,0) =$

a) 0 b) 1 c) 2

d) 3 e) 4 f) 5/2

g) 5 h) 6 I) none of these

4 pts 3. If the power series $\sum a_k (x+3)^k$ converges at x = -1, then:

a) It must converge at x = 7

b) It must converge at x = 0

c) It must converge at both x = -5 and 5

d) It must converge at x = -5

e) It must converge at both –4 and –2

f) It may only converge at x = 3 and x = -1

g) It may only converge at values of x between 0 and –1

h) It must converge at other values of x besides x = -3 but not any of the above

i) Nothing can be determined from the information given other than convergence at x = -1

4 pts 4. The infinite series, $\sum_{k=1}^{\infty} \frac{(-1)^k k^6}{(1.1)^k}$

a) Converges absolutely by the alternating series test

b) Diverges because the limit of the *k*th term does not converge to zero

c) Converges because the limit of the *k*th term does converge to zero

d) Converges by the alternating series test but diverges by the ratio test

e) Diverges by the alternating series test but converges by the root test

f) Converges conditionally by the ratio and the alternating series test

g) Converges absolutely by the root test

h) Converges conditionally by the ratio test

i) This series neither converges nor diverges

4 pts 5. The series, $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges to

a) 1 b) $\frac{3}{2}n$ c) 0

d) $\frac{1}{2n}$ e) $\frac{1}{2}$ f) $\frac{1}{n^2+n}$

g) 3

h) Converges by the ratio test but to none of the above numbers

i) Fails to converge because of the root test

4 pts 6. $\lim_{x\to 0} \frac{e^{-3x}+3x-1}{x^2} =$

a) 0 b) 1 c) 3/2

d) 9/2 e) 9/4 f) -3/2

g) -9/2 h) does not exist

i) exists but is none of the numbers in a) – g)

4 pts 7. Let $f(x) = \ln(\sec(x))$ for $x \in \left[0, \frac{\pi}{4}\right]$. Find the length of the graph of f on this interval.

a) 2 b) $\frac{\pi}{3}$ c) $\frac{7\pi}{2}$

d) $\ln(\sqrt{2\pi})$ e) $\ln(\sqrt{2}\pi) - 1$ f) $\ln(\sqrt{2} + 1)$

g) $\ln(\pi\sqrt{2} + 1)$ h) This curve fails to have finite length

i) The length of the curve exists but is not equal to any of a) – g)

4 pts 8. Find a power series for $\frac{x}{1+x^2}$.

a) $\sum_{n=0}^{\infty} nx^{n+1}$ b) $\sum_{n=0}^{\infty} n(-1)^n x^{n+1}$ c) $\sum_{n=0}^{\infty} (-1)^n x^{2n+1}$

d) $\sum_{n=0}^{\infty} x^{2n+1}$ e) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n}$ f) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{n}$

g) $\sum_{n=0}^{\infty} nx^{2n+1}$ h) $\sum_{n=0}^{\infty} n(-1)^n x^{2n+1}$ i) none of these

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Name _____

Social Security Number _____

Section No. _____ Instructor _____

DO NOT WRITE IN THE SPACE BELOW.

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9. _____

10. (a) _____

    (b) _____

    (c) _____

    (d) _____

11. (a) _____

    (b) _____

    (c) _____

    (d) _____

12. (a) _____

    (b) _____

13. _____

14. (a) _____

    (b) _____

15. _____

16. _____

    Total ____

For Problems 9-16, write answers in the space provided. Please be neat.

9. Find the interval of convergence for $\sum_{n=0}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$. [4 pts]

10. Compute each of the following four integrals. [6 pts each]

(a) $\int \cos 3x \sin 2x\, dx$

(b) $\int x \ln x\, dx$

(c) $\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$

(d) $\int \frac{x^2+2}{x^2+x-2}\,dx$

11. For each of the following series, determine whether it converges or diverges. Justify your answers (e.g., by citing an appropriate convergence test). [4 pts each]

(a) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$

(b) $\sum_{k=1}^{\infty} \frac{k!}{2^{2k}}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n \ln(n+1)}$

12. (a) Sketch the region, R, bounded by the curves $y = x^2$ and $y = x + 2$. [1 pt]

(b) Find the value of $\iint_R xy\,dA$ [4 pts]

13. Use the Trapezoidal Rule to approximate $\int_0^3 \sqrt{x+1}\,dx$ taking subintervals of length 1.

[4 pts]

14. (a) Find the Taylor polynomial $P_3(x)$ of degree 3 for the function $f(x) = 6x^{1/3}$ at (or expanded about) the point $x = 1$. [4 pts]

(b) Use this to approximate $6(1.3)^{1/3}$ [2 pts]

15. Find the area of the surface generated by revolving the curve $y = \sqrt{16 - x^2}$ between $x = 2$ and $x = 2\sqrt{3}$ about the x-axis. [5 pts]

16. Find the centroid of the region bounded by the curves $y = x^2$, $y = -1$, $x = 0$, and $x = 1$. [6 pts]