Name__________________________________

Student Number__________________________

Section Number__________________________

Instructor_______________________________

Math 112 – Fall 2000

Departmental Final Exam

Instructions:

- Be sure to encode you name and student ID number on the bubble sheet.
- Problems 1 through 7 are multiple choice questions. Their answers go on the bubble sheet.
- Write the solutions to problems 8 through 16 directly on the exam paper in the space provided.
- Please write neatly and show all work to receive full credit.
- Work on scratch paper will not be graded.
- Notes, books, and calculators are not allowed.

For administrative use only:

M.C.	/35
8	/12
9	/12
10	/5
11	/6
12	/6
13	/6
14	/6
15	/6
16	/6
Total	/100

Math 112 – Fall 2000

Departmental Final Exam

Part I: Multiple Choice

Problems 1 through 7 are multiple choice. Select the best answer and fill in the corresponding bubble. Please make certain that your name and student number are coded on the bubble sheet.

1. We say $\lim_{x \to a} f(x) = L$ if

   (a) For every $\delta > 0$ there exists $\varepsilon > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

   (b) There exists $\varepsilon > 0$ such that for every $\delta > 0$, if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

   (c) For every $\varepsilon > 0$ there exists $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

   (d) For every $\varepsilon > 0$ and for every $\delta > 0$, if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

   (e) For some $\varepsilon > 0$ there exists $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

   (f) For some $\varepsilon > 0$ and every $\delta > 0$, if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

   (g) I have no idea what $\lim_{x \to a} f(x) = L$ means.

2. Suppose $f'(x) = 2x - 1$ and $f(3) = 4$. Then $f(5) =$

   (a) 9
   (b) 2
   (c) 8
   (d) 12
   (e) 16
   (f) 18
   (g) 22
   (h) 25
   (i) $f(5)$ cannot be determined from the given information.
   (j) None of the above.

3. If $F(x) = \int_x^1 t\sqrt{t^2+1}\,dt$, then $F''(\sqrt{3}) =$

   (a) 0
   (b) $-3/2$
   (c) $-7/2$
   (d) $-2 - \frac{\sqrt{3}}{4}$
   (e) $1/2$
   (f) $-2\sqrt{3}$
   (g) $2\sqrt{3}$
   (h) $2 + 2\sqrt{3}$
   (i) $-2 + \frac{\sqrt{3}}{2}$
   (j) None of the above.

4. $\lim_{x \to 0} \frac{\sin^2(3x)}{5x^2} =$

   (a) 0
   (b) $-\infty$
   (c) $3/5$
   (d) $5/9$
   (e) $25/3$
   (f) ∞
   (g) 1
   (h) $5/3$
   (i) $3/25$
   (j) None of the above.

5. Suppose g and h are the functions defined as follows:

$$g(x) = \begin{cases} x^2+4 & \text{if } x<2, \\ 5 & \text{if } x=2, \\ x^3 & \text{if } x>2, \end{cases} \quad \text{and} \quad h(x) = \begin{cases} x^2+5 & \text{if } x<2, \\ 9 & \text{if } x=2, \\ 1+x^3 & \text{if } x>2. \end{cases}$$

Only one of the following statements is correct. Which one?

(a) Neither $g(x)$ nor $h(x)$ is defined at $x = 2$.

(b) Both $g(x)$ and $h(x)$ are continuous at $x = 2$.

(c) $g(x)$ is not continuous at $x = 2$, and $h(x)$ is differentiable at $x = 2$.

(d) $g(x)$ has a jump discontinuity at $x = 2$, and $h(x)$ has a removable discontinuity at $x = 2$.

(e) $g(x)$ is continuous at $x = 2$, and $h(x)$ has a removable discontinuity at $x = 2$.

(f) $g(x)$ is not continuous at $x = 2$, and $h(x)$ is continuous at $x = 2$.

(g) $g(x)$ is not continuous at $x = 2$, and $h(x)$ has a removable discontinuity at $x = 2$.

(h) Neither $g(x)$ nor $h(x)$ is continuous at $x = 2$.

6. $\displaystyle\int_0^2 \frac{dx}{4+x^2} =$

(a) $\pi/2$
(b) $\pi/8$
(c) 0
(d) $\sqrt{2}$
(e) $1/\sqrt{2}$
(f) $\pi/4$
(g) 2π
(h) $\frac{\sqrt{3}}{2}$
(i) $\pi/3$
(j) None of the above.

7. $\displaystyle\int_0^{\ln 3} e^{-x}\,dx =$

(a) $1 - \ln(3)$
(b) 3
(c) $-2/3$
(d) 1
(e) $-1/3$
(f) $e^{\ln(3)} - 1$
(g) 2
(h) $2/3$
(i) $1/3$
(j) None of the above.

Part II: Written Solutions

Neatly write answers to each of the following questions in the space provided. Show your work.

8. Evaluate the following derivatives:

   (a) $\dfrac{d}{dx}\sqrt{1+\sin(x)}$

   (b) $\dfrac{d}{dx}\big(\,x^2\ln(3x+1)\,\big)$

   (c) $\dfrac{d}{dx}\dfrac{\tan(3x)}{x}$

9. Evaluate the following indefinite integrals:

   (a) $\displaystyle\int \sqrt{3x+5}\,dx$

   (b) $\displaystyle\int \sin(2\pi x)\,dx$

   (c) $\displaystyle\int \frac{\sec^2(2x)}{4-\tan(2x)}\,dx$

10. Use implicit differentiation to find dy/dx if

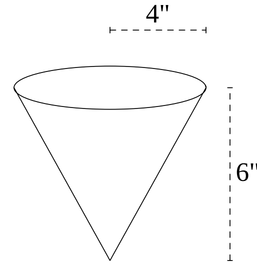
$$2xy - y^3 + 1 = x + 2y.$$

11. (a) State the definition of the derivative $f'(x)$ of the function $f(x)$.

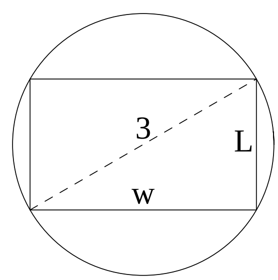
(b) Use the definition of the derivative to compute $f'(x)$ for the function $f(x) = 5x - x^2$. [To receive credit you must use the definition.]

12. A conical paper cup 8 inches across the top and 6 inches deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep? Express the answer in inches per minute.

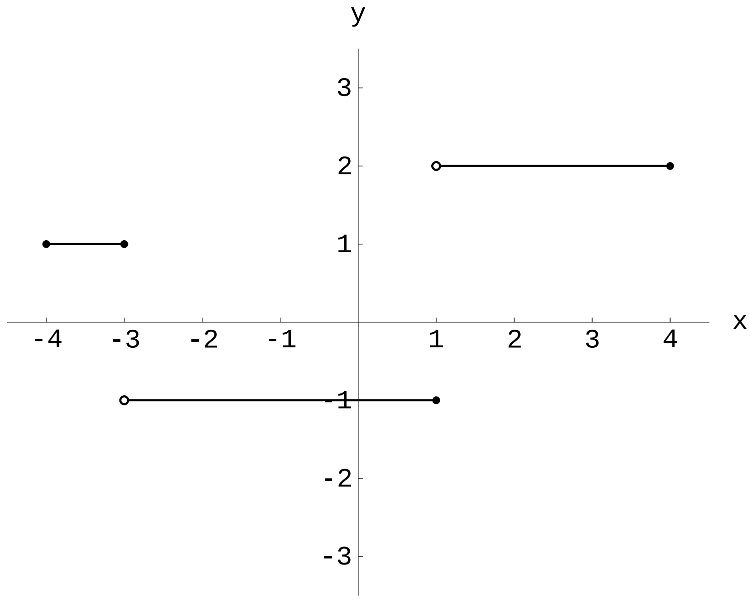
[Note: The formula for the volume of a cone of base radius r and height h is $\frac{1}{3}\pi r^2 h$.]



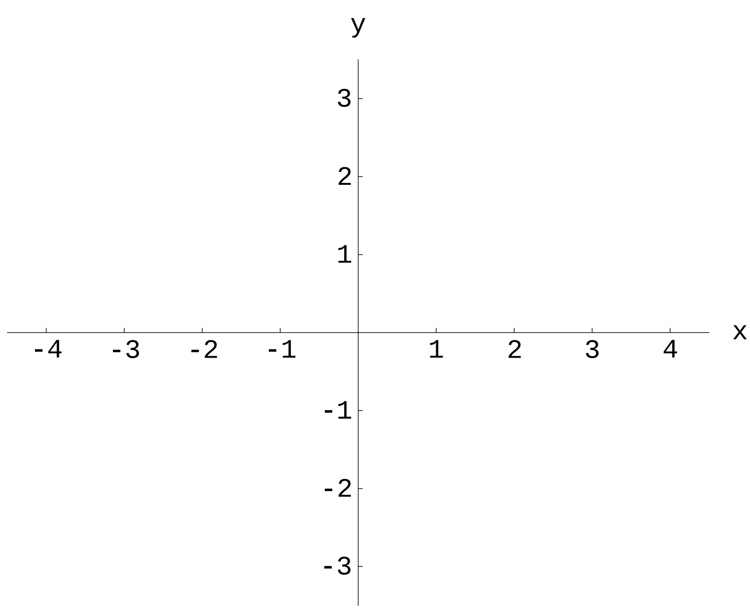
13. The cross section of a beam is in the form of a rectangle of length L and width w. Assuming that the strength of the beam varies directly with $w^2 L$, what are the dimensions of the strongest beam that can be sawed from a round log of diameter 3 feet?



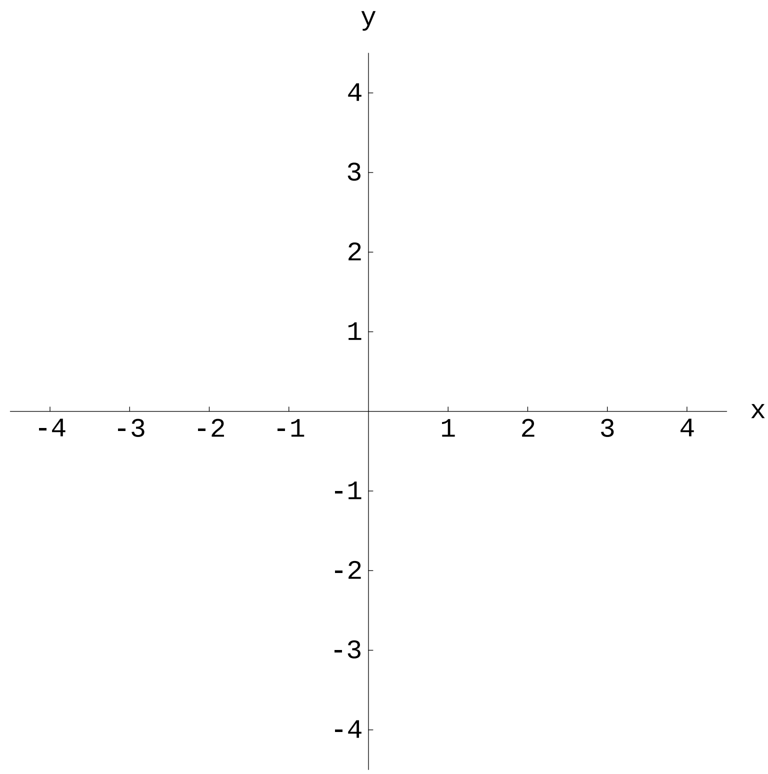
14. The graph of a piecewise linear function $y = g(x)$ on the interval $-4 \leq x \leq 4$ is sketched below.



On the set of axes below, draw a sketch of the graph of $F(x) = \int_{-2}^{x} g(t)\,dt$.



15. (a) On the axes below sketch the region bounded by the curves $x = y^2$ and $x - y = 2$.
[Note: There are two points of intersection; they are $(1, -1)$ and $(4, 2)$]



(b) Set up an integral ***with respect to*** y that represents the area A of the bounded region. (Do NOT evaluate the integral.)

16. The number of bacteria in a certain culture is increasing at a rate proportional to the number present. Suppose there are 1000 bacteria initially (when $t = 0$), and that 1500 are present after 2 hours (when $t = 2$).

(a) Let $P(t)$ represent the number of bacteria present at time t. Find a formula for $P(t)$.

(b) How long will it take for the number of bacteria to double?