Math 113 – Fall 2001

Departmental Final Exam

Part I: Multiple Choice

Problems 1 through 8 are multiple choice. Select the best answer and fill in the corresponding bubble. Please make certain that your name and student number are coded on the bubble sheet. Each multiple choice problem is worth 3 points.

1. Which trigonometric substitution should be made in order to evaluate the integral $\displaystyle\int \frac{\sqrt{x^2+9}}{x}\,dx$?

   (a) $x = 3\tan u$ (b) $x = 9\sin u$ (c) $x = 3\sin u$
   (d) $x = 9\tan u$ (e) $x = 3\sec u$ (f) $x = 9\sec u$

2. Which integral represents the volume of the solid generated by rotating the region bounded by the curves $y = \sqrt{x}$ and $y = x^3$ about the y-axis?

   (a) $\displaystyle\int_0^1 \pi\left[(\sqrt{x})^2 - (x^3)^2\right]\,dx$
   (b) $\displaystyle\int_0^1 \pi x\left[\sqrt{x} - x^3\right]\,dx$
   (c) $\displaystyle\int_0^1 2\pi x\left[\sqrt{x} - x^3\right]\,dx$
   (d) $\displaystyle\int_0^1 \pi x\left[\sqrt{x} - x^3\right]^2\,dx$
   (e) $\displaystyle\int_0^1 \pi\left[(x^3)^2 - (\sqrt{x})^2\right]\,dx$
   (f) $\displaystyle\int_0^1 \pi x\left[x^3 - \sqrt{x}\right]\,dx$
   (g) $\displaystyle\int_0^1 2\pi x\left[x^3 - \sqrt{x}\right]\,dx$
   (h) $\displaystyle\int_0^1 2\pi x\left[x^3 - \sqrt{x}\right]^2\,dx$

3. What is the form of the partial fraction decomposition of $\dfrac{2x+3}{x(x^2+4)^2}$?

   (a) $\dfrac{A}{x} + \dfrac{B}{(x^2+4)^2}$
   (b) $\dfrac{A}{x} + \dfrac{B}{x^2+4} + \dfrac{C}{(x^2+4)^2}$
   (c) $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+4} + \dfrac{Dx+E}{x^2+4}$
   (d) $\dfrac{A}{x} + \dfrac{Bx+C}{(x^2+4)^2}$
   (e) $\dfrac{A}{x} + \dfrac{Bx+C}{x^2+4} + \dfrac{Dx+E}{(x^2+4)^2}$

4,5. Fill in the blanks in the following statement to make it correct. The expression you put in the first blank will be the answer to (4), and the expression you put in the second blank will be the answer to (5).

"We say $\lim_{n\to\infty} a_n = L$ if and only if, for each $\epsilon > 0$, there exists a positive integer K such if _____ then _____."

   (a) $|a_n - n| < \epsilon$ (b) $|a_n - \epsilon| < L$ (c) $|a_n - L| < K$
   (d) $|a_n - K| < L$ (e) $|a_n - L| < \epsilon$ (f) $|a_n - \epsilon| < K$
   (g) $n > K$ (h) $K > n$ (i) $n > \epsilon$
   (j) $K > \epsilon$

6. The limit of the sequence $\left\{\left(1+\dfrac{x}{n}\right)^{3n}\right\}$ is

(a) x^3 (b) 3^x (c) $3x$
(d) 3 (e) $\ln(3)$ (f) e^{3x}
(g) $1/n$ (h) e^x (g) $\ln(1+\frac{x}{n})$
(i) $\frac{1}{3x}$

7. Which integral represents the length of the graph of $f(x) = x^2$ from $x = 1$ to $x = 3$?

(a) $\displaystyle\int_1^3 x^2\,dx$ (f) $\displaystyle\int_1^3 (1+2x)dx$

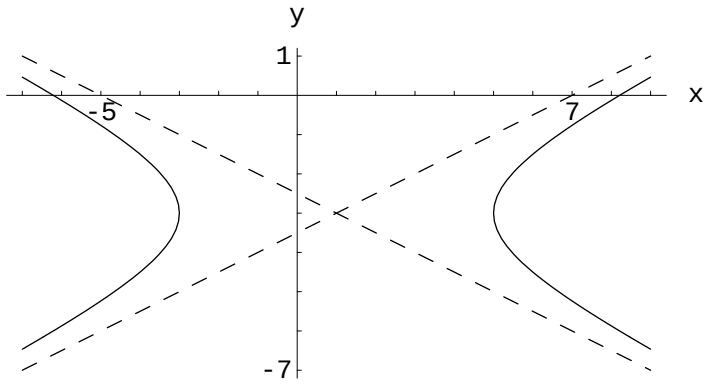
(b) $\displaystyle\int_1^3 \sqrt{1+4x^2}dx$ (g) $\displaystyle\int_1^3 (1+\sqrt{2x})dx$

(c) $\displaystyle\int_1^3 \sqrt{1+2x}dx$ (h) $\displaystyle\int_1^3 \sqrt{1+4x}dx$

(d) $\displaystyle\int_1^3 \sqrt{1+x^2}dx$ (i) $\displaystyle\int_1^{\sqrt{3}} (1+x^2)dx$

(e) $\displaystyle\int_1^3 2\pi x^2 dx$

8. Find an equation for the following graph:



(a) $\dfrac{(x-1)^2}{16}+\dfrac{(y+3)^2}{4}=1$ (f) $\dfrac{(y-3)^2}{4}-\dfrac{(x-1)^2}{16}=1$

(b) $\dfrac{(x-3)^2}{16}-\dfrac{(y-1)^2}{4}=1$ (g) $\dfrac{(x-1)^2}{16}-\dfrac{(y+3)^2}{4}=1$

(c) $\dfrac{(x-3)^2}{16}+\dfrac{(y-1)^2}{4}=1$ (h) $\dfrac{(x-1)^2}{16}+\dfrac{(y-3)^2}{4}=1$

(d) $\dfrac{(y+3)^2}{4}-\dfrac{(x-1)^2}{16}=1$ (i) $\dfrac{(x-1)^2}{16}-\dfrac{(y-3)^2}{4}=1$

(e) $\dfrac{(x-1)^2}{4}+\dfrac{(y+3)^2}{16}=1$

Part II: Written Solutions

For problems 9 - 17, write your answers in the space provided. Neatly show your work for full credit.

9. (6 points) Find the volume of the solid whose base is the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (a > 0 \text{ and } b > 0)$$

and whose cross sections perpendicular to the x-axis are squares.

10. (5 points) A vertical cylindrical tank of radius 3 feet and height 8 feet is half full of oil weighing 60 pounds per cubic foot. Write an integral that represents the work done in pumping the oil to a valve at the top of the tank.

[Do NOT evaluate the integral.]

11. Evaluate the following indefinite integrals.

(a) (5 points) $\int x^2 \ln(x)\, dx$

(b) (5 points) $\int x \sin(x)\, dx$

(c) (5 points) $\int \dfrac{dx}{x(x+3)}$

(d) (7 points) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$

12. (6 points) Determine the interval of convergence for the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k-1}x^k}{k2^k}$.

13. (4 points) Determine the value of $\displaystyle\lim_{x\to 0} \frac{2^x-1}{x}$.

14. (6 points) Find the area of the region that consists of all points that lie within the circle $r = 2\cos\theta$ but outside the circle $r = 1$.

15. (6 points) Use the 8th degree Taylor polynomial for $x^2 \sin(x^2)$ to estimate the value of the integral

$$\int_0^1 x^2 \sin(x^2)\, dx.$$

16. For each series tell whether it converges or diverges and justify your answer by applying an appropriate convergence test.

(a) (5 points) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{k^3+1}$

(b) (5 points) $\displaystyle\sum_{k=3}^{\infty} \cos(1/k^3)$

(c) (5 points) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$

17. (6 points) Determine which of the following functions is smallest and which one is largest for very small positive x.

$$f(x) = e^{2x} \qquad g(x) = \frac{1}{1-2x} \qquad h(x) = \frac{1}{(1-x)^2}$$

[Hint: Look at the first few terms in the Taylor series for each function.]