

Name_____

Student Number_____

Section Number_____

Instructor_____

Math 113 – Winter 2001

Departmental Final Exam

Instructions:

- Be sure to encode you name and student ID number on the bubble sheet.
- Problems 1 through 7 are multiple choice questions. Their answers go on the bubble sheet.
- Write the solutions to problems 8 through 17 directly on the exam paper in the space provided.
- Work on scratch paper will not be graded.
- Please write neatly and show your work to receive full credit.
- Notes, books, and calculators are not allowed.

For administrative use only:

| | |
|-------|------|
| M.C. | /35 |
| 8 | /7 |
| 9 | /6 |
| 10 | /8 |
| 11 | /8 |
| 12 | /6 |
| 13 | /6 |
| 14 | /6 |
| 15 | /6 |
| 16 | /6 |
| 17 | /6 |
| Total | /100 |

Math 113 – Winter 2001

Departmental Final Exam

PART I: MULTIPLE CHOICE

Problems 1 through 7 are multiple choice. Select the best answer and fill in the corresponding bubble. Please make certain that your name and student number are coded on the bubble sheet.

- The approximation to the integral $\int_0^1 x^2 dx$ obtained by using the Trapezoid Rule with 2 equally sized subintervals is
 - 3/4
 - 5/4
 - 1/8
 - 3/8
 - 5/8
 - 7/8
 - 1/16
 - 3/16
 - 5/16
 - 7/16
- For which real numbers does the series $\sum_{k=5}^{\infty} \frac{kx^k}{2^k}$ converge?
 - The series converges only if $x = 0$.
 - $x \in (-1, 1)$
 - $x \in (0, 1)$
 - $x \in (0, 2)$
 - $x \in (-2, 0)$
 - $x \in (-2, 2)$
 - The series converges for all real values of x .
 - The series converges for some values of x but it can't be decided what those values are.
 - The series does not converge for any values of x .
- If $f(x, y) = y \cos(x - y)$, what is $f_y(2, 2)$?
 - 2
 - 1
 - 0
 - 1
 - 2
 - 3
 - $\cos(2) + 2 \sin(2)$
 - $\cos(2) - \sin(2)$
 - $\sin(2) + 2 \cos(2)$
 - answer not given

4. What is the value of $\int_0^{\infty} xe^{-x} dx$?

- | | |
|--------------|----------------------|
| (a) 0 | (f) $e - 1$ |
| (b) 1 | (g) $1 - e^{-1}$ |
| (c) -1 | (h) ∞ |
| (d) e | (i) answer not given |
| (e) e^{-1} | |

5. What is the value of the limit $\lim_{x \rightarrow 0^+} (3x)^x$?

- | | |
|--------------|-------------------------------|
| (a) 0 | (f) The limit does not exist. |
| (b) 1 | (g) $\ln(3)$ |
| (c) ∞ | (h) $e^{1/3}$ |
| (d) 3 | (i) e^3 |
| (e) $1/3$ | (j) answer not given |

6. Evaluate $\int_0^{\pi/6} \sin^2(3x) dx$.

- | | |
|---------------|----------------------|
| (a) 0 | (f) $\pi/4$ |
| (b) 1 | (g) $1/3$ |
| (c) $\pi/6$ | (h) $1/4$ |
| (d) $1/2$ | (i) $\pi/12$ |
| (e) $\pi^2/6$ | (j) answer not given |

7. The value of $\int_0^{\infty} \frac{dx}{4+x^2}$ is

- | | |
|------------------|--|
| (a) $\ln(4+x^2)$ | (f) e^2 |
| (b) π | (g) e^{-2} |
| (c) $\pi/2$ | (h) $4e^{-2}$ |
| (d) $\pi/4$ | (i) 1 |
| (e) $\pi/8$ | (j) This integral is undefined since the upper limit is ∞ . |

PART II: WRITTEN SOLUTIONS

For problems 8 - 20, write your answers in the space provided. Neatly show your work for full credit.

8. Find the volume V of the solid generated by revolving the region between

$$y = x^2 \quad \text{and} \quad y = 2x$$

about the x -axis.

9. Determine the partial fraction decomposition of the function $R(x) = \frac{1}{x(x^2 + 1)}$.

10. Find the following antiderivatives.

(a) $\int e^x \cos(3x) dx$

(b) $\int \cos^3(x) dx$

11. Evaluate the following definite integrals.

(a) $\int_0^1 \frac{dx}{(5-x^2)^{3/2}}$

(b) $\int_1^e x^2 \ln(x) dx$

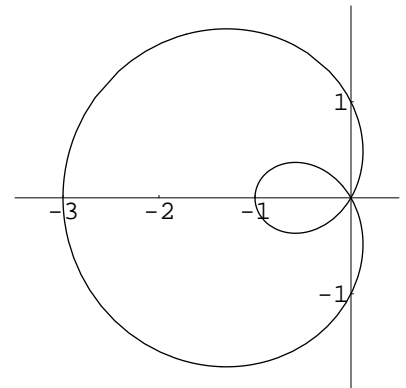
12. A tank filled with water is in the shape of the solid generated by revolving the parabola $y = \frac{3}{4}x^2$ for $0 \leq x \leq 4$ around the y -axis. The dimensions of the tank are measured in feet. Assume water weighs 62.5 pounds per cubic foot. Write an integral representing the work done in pumping the water to a outlet 1 foot above the top of the tank.

Do NOT evaluate the integral.

13. Write an expression involving integration that represents the area of the region between the inner and outer loops of the limaçon

$$r = 1 - 2 \cos(\theta).$$

(Do NOT evaluate the expression. Leave the integrals unevaluated.)



14. For each series tell whether it converges and justify your answer by citing an appropriate convergence test.

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2 + 3}}$$

(b)
$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$$

(c)
$$\sum_{n=1}^{\infty} \frac{n+2}{n^3-7}$$

15. Expand $g(x) = xe^{x^2}$ as a series in powers of x . Please express your answer using summation notation.

[Hint: Think about the series for e^x .]

16. Determine the Taylor series for the function $f(x) = \ln(x)$ in terms of powers of $x - 1$. Please express your answer in summation notation.

17. Evaluate $\iint_{\Omega} (x^4 - 2y) dx dy$ where Ω is the region bounded by the lines $x = -1$ and $x = 1$ and the curves $y = x^2$ and $y = -x^2$ as in the figure.

