

RED
TIMED

Name _____

Student Number _____

Section Number _____

Instructor _____

Math 113 – Winter 2002 – Form A

Departmental Final Exam

Instructions:

- The time limit is 3 hours. There is a 10 minute grace period for turning your exam in. After that you will lose 1 point for each minute the exam is late.
- Be sure to encode your name and student ID number on the bubble sheet.
- Problems 1 through 8 are multiple choice questions. Their answers go on the bubble sheet.
- Write the solutions to problems 9 through 17 directly on the exam paper in the space provided.
- Work on scratch paper will not be graded.
- Please write neatly and show your work to receive full credit.
- Notes, books, and calculators are not allowed.

For administrative use only:

M.C.	/24
9	/7
10	/9
11(a)	/5
11(b)	/6
11(c)	/6
12	/4
13	/6
14	/5
15	/7
16	/6
17(a)	/5
17(b)	/5
17(c)	/5
Total	/100

Math 113 – Winter 2002

Departmental Final Exam
(Form A)

PART I: MULTIPLE CHOICE

Problems 1 through 8 are multiple choice. Select the best answer and fill in the corresponding bubble. Please make certain that your name and student number are coded on the bubble sheet. Each multiple choice problem is worth 3 points.

1. Which integral represents the length of the graph of $f(x) = \sin(x)$ from $x = 0$ to $x = \pi$?

(a) $\int_0^\pi \sin x \, dx$

(f) $\int_0^\pi \sqrt{1 + \sin^2 x} \, dx$

(b) $\int_0^\pi \cos^2 x \, dx$

(g) $\int_0^\pi \sqrt{1 + \cos^2 x} \, dx$

(c) $\int_0^\pi \sqrt{\sin x} \, dx$

(h) $\int_0^\pi (1 + \cos^2 x) \, dx$

(d) $\int_0^\pi \sqrt{1 + \cos x} \, dx$

(i) $\int_0^\pi \sqrt{1 - \cos^2 x} \, dx$

(e) $\int_0^\pi \sin^2 x \, dx$

(j) $\int_0^\pi (1 + \sin^2 x) \, dx$

2. If the sequence $\{a_n\}$ converges, then

(a) the sequence $\{\frac{a_n}{n}\}$ converges to 0.

(b) the sequence $\{\frac{a_n}{n}\}$ converges but the limit cannot be determined.

(c) the sequence $\{\frac{a_n}{n}\}$ diverges.

(d) it cannot be determined whether or not $\{\frac{a_n}{n}\}$ converges because a_n is not given.

3. Evaluate the integral $\int_0^\pi \cos^2(x) \, dx$.

(a) $-\frac{2}{3}$

(b) 0

(c) $\frac{1}{2}$

(d) $\frac{\pi}{2}$

(e) $\pi + 1$

(f) $\frac{\pi}{2} + 1$

(g) π

(h) 1

(i) 2

(j) 2π

4. If the trapezoid rule with $n = 10$ is used to approximate $\int_1^2 \sqrt{1+x^2} dx$, we obtain the expression

$$\frac{1}{20} \left(\sqrt{2} + 2\sqrt{2.21} + \cdots + \sqrt{5} \right).$$

The first term in the sum in parentheses is $\sqrt{2}$, the second term is $2\sqrt{2.21}$, \dots . What is the sixth term?

- (a) 3 (b) 3.2 (c) 3.4 (d) $\sqrt{3.56}$ (e) $2\sqrt{3.56}$
 (f) $4\sqrt{3.56}$ (g) $\sqrt{3.25}$ (h) $2\sqrt{3.25}$ (i) $4\sqrt{3.25}$

5. The region bounded by the curves $y = x^3$, $y = x^2$, and the line $x = 2$ is revolved about the x -axis. Which integral represents the volume of the resulting solid?

- (a) $2\pi \int_1^2 x(x^3 - x^2) dx$ (b) $\pi \int_1^2 (x^3 - x^2)^2 dx$ (c) $\pi \int_1^2 (x^3 - x^2) dx$
 (d) $2\pi \int_1^2 x(x^6 - x^4) dx$ (e) $\pi \int_1^2 (x^6 - x^4)^2 dx$ (f) $\pi \int_1^2 (x^6 - x^4) dx$

6. Fill in the blank. We say that $\lim_{n \rightarrow \infty} a_n = L$ if and only if for every $\epsilon > 0$, there is a positive integer K such that _____ whenever $n \geq K$.

- (a) $|a_n - n| < \epsilon$ (b) $|a_K - \epsilon| < L$ (c) $|a_n - L| < K$ (d) $|a_n - K| < L$
 (e) $|a_n - L| < \epsilon$ (f) $|a_n - \epsilon| < K$

7. $\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

- (a) converges absolutely.
 (b) converges by the p -test, but diverges by the alternating series test.
 (c) converges conditionally by the ratio test.
 (d) converges conditionally.
 (e) converges by the divergence test.
 (f) diverges.
 (g) it cannot be determined whether the series converges or not since there is no x .

8. The series

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

is the Taylor series for the function

- (a) $\cos x$ (b) $\ln x$ (c) $\sin x$ (d) x
 (e) $\frac{1}{1-x}$ (f) e^x (g) $\frac{1}{1+x}$ (h) $\sinh x$

PART II: WRITTEN SOLUTIONS
(Form A)

For problems 9 - 17, write your answers in the space provided. Neatly show your work for full credit.

9. (7 points) Find the volume of the solid generated when the region bounded by the x -axis, the line $x = 2$, and $y = \ln(x)$ is revolved about the y -axis.

10. (9 points) Find the centroid (\bar{x}, \bar{y}) of the region in the first quadrant that is bounded by the y -axis, the line $y = 1$, and the curve $y = x^2$.

11. Evaluate the following indefinite and definite integrals.

(a) (5 points) $\int x e^{3x} dx$

(b) (6 points) $\int \frac{x^2}{\sqrt{4-x^2}} dx$

(c) (6 points) $\int \sin^{-1}(x) dx$

12. (4 points) What is the form of the partial fraction decomposition of the following expression?

$$\frac{8x^3 - 17x + 24}{(x - 1)^2(x^2 + 1)^2}$$

[Your answer should have symbolic constants. Do NOT try to determine the values of the constants.]

13. (6 points) Find the Taylor polynomial of degree 3 for $f(x) = \sqrt{x}$ centered at $a = 1$.

14. (5 points) Write the equation

$$(x^2 + y^2)^2 = x^2 - y^2$$

in polar coordinates and simplify your answer.

15. (7 points) Evaluate the improper integral $\int_1^{\infty} \frac{1}{x(x+1)} dx$.

16. (6 points) The equation $y^2 + \cos(y) = \frac{3}{2}$ is difficult to solve exactly for y . Solve it approximately for y by replacing $\cos(y)$ by its Taylor series at 0 and then disregarding any powers of y higher than y^2 .

17. For each series tell whether it converges or diverges and justify your answer by applying an appropriate convergence test.

(a) (5 points) $\sum_{k=1}^{\infty} \frac{k^2}{3^k}$

(b) (5 points) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$

(c) (5 points) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 + k}$