Name______________________________

Student Number________________________

Section Number________________________

Instructor____________________________

Math 112 – Winter 2005

Departmental Final Exam

Instructions:

- The time limit is 3 hours.
- Problems 1 through 8 are multiple choice questions, each worth 4 points.
  Their answers **MUST** be entered on the grid on page 1
- Write solutions to problems 9 through 19 on the exam paper in the space provided.
  Problems 9, 10, 12, 15, 16, 17, 18 worth 6 points each, and Problem 19 is 8 points.
- Work on scratch paper will not be graded.
- Please write neatly, and simplify your answers.
- Notes, books, and calculators are not allowed.
- Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

For administrative use only:

M.C.	/32
9	/6
10	/6
11	/6
12	/6
13	/6
14	/6
15	/6
16	/6
17	/6
18	/6
19	/8
Total	/100

Math 112 – Winter 2005

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Part I: Multiple Choice

Problems 1 through 8 are multiple choice. Each multiple choice problem is worth 4 points. In the grid below fill in the square corresponding to each correct answer.

1	A	B	C	D	E	F	G	H	I	J
2	A	B	C	D	E	F	G	H	I	J
3	A	B	C	D	E	F	G	H	I	J
4	A	B	C	D	E	F	G	H	I	J
5	A	B	C	D	E	F	G	H	I	J
6	A	B	C	D	E	F	G	H	I	J
7	A	B	C	D	E	F	G	H	I	J
8	A	B	C	D	E	F	G	H	I	J

1. If $f(t) = 3t^3 - 4t^2 + 5t - 6$ is the position of a particle at time t, find the average velocity of the particle for $1 \leq t \leq 3$.

   (a) $\frac{61}{2}$ (b) $\frac{81}{2}$ (c) $\frac{1}{2}$ (d) 54 (e) 28

   (f) 31

2. Given $\lim_{t \to 3} \frac{t^2 - 9}{t - 3} = 6$, for $\epsilon = \frac{1}{10}$, find the largest possible δ.

   (a) $\frac{1}{10}$ (b) $\frac{1}{100}$ (c) 100 (d) 10 (e) $\frac{6}{100}$

   (f) $\frac{6}{10}$ (g) $\frac{3}{100}$ (h) $\frac{3}{10}$

3. For $f(x) = \dfrac{4x+1}{2x-5}$, find the vertical asymptote.

(a) $-\dfrac{1}{4}$ (b) 2 (c) 5 (d) 4 (e) $\dfrac{5}{2}$ (f) no vertical asymptotes

4. $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2}$ is equal to

(a) $\displaystyle\int_0^\infty \frac{dx}{x}$ (b) $\displaystyle\int_0^1 \frac{dx}{x}$ (c) $\displaystyle\int_0^1 x\,dx$ (d) $\displaystyle\int_0^\infty \frac{K}{x}\,dx$

(e) $\displaystyle\int_0^1 \frac{K}{x}\,dx$ (f) $\displaystyle\int_0^1 x^2 dx$

5. Find the average value of $f(x) = 4x - x^2$ on $[0, 4]$.

(a) 0 (b) 8 (c) -8 (d) $\dfrac{8}{3}$ (e) $\dfrac{13}{4}$ (f) $\dfrac{32}{3}$

6. $f(x) = e^{\sin^{-1} x}$. Evaluate $f'\left(\dfrac{\sqrt{3}}{2}\right)$.

(a) $\dfrac{4}{3}e^{\frac{\pi}{6}}$ (b) $\dfrac{3}{4}e^{\frac{\pi}{6}}$ (c) $\dfrac{\sqrt{3}}{2}e^{\frac{\pi}{6}}$ (d) $4e^{\frac{\pi}{3}}$ (e) $2e^{\frac{\pi}{3}}$

(f) $\dfrac{2}{\sqrt{3}}e^{\frac{\pi}{6}}$ (g) $\dfrac{1}{2}e^{\frac{\pi}{3}}$ (h) $e^{\frac{\pi}{3}}$

7. The function $f(x) = x^3 - 6x^2 + 9x + 3$

(a) has no local extreme points and no inflection points

(b) has local maximum at $x = 1, 3$ and inflection at $x = 2$

(c) has local minimum at $x = 1, 3$ and inflection at $x = 2$

(d) has local minimum at $x = 1$, local maximum at $x = 3$, and inflection at $x = 2$

(e) has local maximum at $x = 1$, local minimum at $x = 3$, and inflection at $x = 2$

(f) has local maximum at $x = 1$, local minimum at $x = 3$, and no inflection points

8. If $f(x) = x^x$, then $f'(2) =$

(a) $4(1 + \ln 2)$ (b) $1 + \ln 2$ (c) $1 + \ln x + C$ (d) $x^x + \ln 2$

(e) 1 (f) $\ln 16$.

The answers to the multiple choice MUST be entered on the grid on page 1. Otherwise, you will not receive credit.

Part II: Written Solutions

For problems 9 - 19, write your answers in the space provided. Neatly show your work for full credit.

9. Assume that the equation $x^2 - 2xy - y^2 = 0$ implicitly defines y as a function of x. Find y''.

10. $f(x) = 4x - x^2$ on $[0, 4]$. Using the partition $[0, 1, 2, 3, 4]$, find

    (i) the right-hand sum (ii) the trapezoidal approximation

11. A kite is flying at an angle of elevation of 30°. The kite string is being taken in at the rate of $\frac{1}{2}$ foot per second. If the angle of elevation does not change, how fast is the kite losing altitude?

12. Find the limit. If the limit does not exist, write "Limit = ∞," or "Limit = −∞," or *if neither of these is true,* write "Limit doesn't exist."

(*a*) $\lim_{x\to 0} \frac{\sec^2 x - 1}{x^2}$

(*b*) $\lim_{x\to\infty} \left(\sqrt{x^2+3x} - x\right)$

13. A certain tree grows in height at a rate inversely proportional to its height. If it grows 3 feet in the first 3 years, how tall will it be at the end of 4 years? (Assume that the tree initially has a height of 0 feet.)

14. Find the following derivatives:

$(a)\dfrac{d}{dx}\left(e^{-3x}\cos 3x\right)$

(b)Find $h'(0)$, where $h(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases}$

15. Compute the following integrals:

(a) $\int \frac{e^x}{1+e^x}\,dx$

(b) $\int x^3\sqrt{9-x^2}\,dx$

16. $f(x) = \frac{\tan 4x}{\sin 2x}$ is undefined at $x = 0$, therefore it is discontinuous at $x = 0$. Can $f(x)$ be defined at $x = 0$ so that it is continuous there? If not, why not? If so, how so, and what is the defined value?

17. A drop of water is placed in a petri dish for culturing. After one day, it is estimated that 400 bacteria are present. After 2 days, it is estimated that 1800 bacteria are present. Assuming exponential growth, how many bacteria were present in the original drop of water?

18. In a particular apartment complex of 80 units, it is found that all units remain occupied when the rent is $400 per month. For each $20 increase in the rent, one unit becomes vacant, on the average. Occupied units require $40 per month for maintenance, while vacant units require none. Fixed costs for the complex are $24,000 per month. What rent should be charged for maximum profit?

19. a) State The Fundamental Theorem of Calculus as given in Chapter 6 of the text.

b) Use Rolle's Theorem to prove that the function $f(x) = 4 - x - 6x^3$ has only one real zero.