

# 1 Pretest Review

The pretest will consist of 20 problems, each of which is similar to one of the following 49 problems. If you can do problems like these 49 listed below, you will have no problem with the pretest.

## Values of trig. functions at reference angles

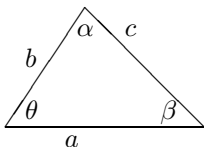
1. Find  $\cos\left(-\frac{1}{6}\pi\right)$ .
2. Find  $\csc\left(\frac{5}{8}\pi\right)$ .
3. Find  $\sin\left(\frac{2}{3}\pi\right)$ .
4. Find  $\tan\left(\frac{11}{6}\pi\right)$ .
5. Find  $\sec\left(\frac{7}{6}\pi\right)$ .
6. Find  $\cot\left(\frac{4}{3}\pi\right)$ .

## Trig identities

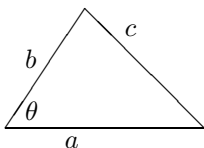
7. Give the formula for  $\cos(x - y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .
8. Give the formula for  $\cos(2x)$  in terms of  $\cos x$  and  $\sin x$ .
9. Give the formula for  $\cos(x + y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .
10. Give the formula for  $\sin(x - y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .
11. Give the formula for  $\sin(x + y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .
12. Give the formula for  $\sin(2x)$  in terms of  $\cos x$ ,  $\sin x$ .

## Law of cosines and sines

13. In the picture,  $\theta = \frac{1}{4}\pi$ ,  $\alpha = \frac{2}{3}\pi$  and  $c = 2$ . Find  $a$ .



14. In the picture,  $a = 5$ ,  $b = 3$ , and  $\theta = \frac{1}{6}\pi$ . Find  $c$ .



## Inverses

15. Let  $f(x) = \frac{3x-7}{11x+5}$ . Find  $f^{-1}(x)$ .
16. Suppose  $rx^2 + sx + t = 0$  where  $r \neq 0$ . Then  $x =$
17. Find all solutions to  $x^4 - 11x^2 + 30 = 0$ .
18. Find all solutions to  $x^2 + 5x + 6 = 0$
19. Suppose  $-2x + y = -4$  and  $x - 4y = -4$ . Find  $x - y$ .
20. For what value of  $k$  does the system of equations,  $2x + y = 1$ ,  $12x + ky = 12$  have no solution?

## Word problems

21. A water tank is initially  $\frac{1}{4}$  full. After adding 14 gallons of water, it is  $\frac{7}{8}$  full. What is the capacity of the tank in gallons?
22. The perimeter of a rectangle is 9 times its width. If the length of the rectangle is 49, find its width.

## Solving inequalities

23. Solve  $x(x + 5)(x - 4) < 0$  for  $x$ .
24. Solve  $|5x - 25| < 20$  for  $x$ .
25. Find the solution to the inequality,  $|4x - 2| < 6$

## Solving Equations

26. Let  $y = \frac{7x+5}{3x-7}$ . Solve for  $x$ .
27. Solve for  $x$  in the equation  $10(1 + 17x)^{1/2} + 85x(1 + 17x)^{-1/2} = 0$ .
28. Find all positive solutions to  $\sqrt{(x^2 + 16)} = 6$ .

## Simplifying expressions

29. Simplify  $(3^4)^{-\frac{1}{4}}$ .
30. Simplify  $\frac{2xy^3}{2^{-2}xy^2}$ .  
Simplifying
31. Simplify  $\frac{1}{x^2+4} + \frac{2}{x}$ .
32. Simplify  $\frac{1}{x^2-4} + \frac{2}{3x-6}$ .
33. Simplify  $|\sqrt{7} - 7| - |-5|$ .
34. Simplify  $|\frac{1}{8} - \frac{1}{4}\sqrt{2}| - |-4|$ .  
Simplify difference quotient
35. Suppose  $f(x) = x^2 + 6x$ . Simplify  $\frac{f(x+h)-f(x)}{h}$ .

## Factoring of polynomials

36. Factor the polynomial  $x^2 + 13x + 42$ .
37. Factor the polynomial  $6x^2 + 43x + 42$ .

### Composition Of Functions

38. Suppose  $f(-2) = 3$ ,  $f(3) = -2$ , and  $f(1) = -1$  while  $g(3) = 1$ ,  $g(-2) = 2$ , and  $g(1) = 1$ . Find  $f(g(-2+3))$ .
39. Let  $f(x) = -3x^2 + 2x$  and let  $g(x) = -3x^2$ . Find  $f(g(1))$ .

### Lines

40. A line having the equation  $y = mx + b$  passes through the points  $(-4, -7)$  and  $(9, 1)$ . Find  $m + b$ .
41. Find the equation of the line through the points  $(-4, 3)$ ,  $(5, 4)$ .
42. Find the equation of the line perpendicular to  $-18x + 6y = 6$  through  $(0, 6)$ .
43. Find the equation of the line parallel to  $-3x + y = 3$  and passing through  $(0, 8)$ .
44. A line has the equation  $6x = 5y + 10$ . What is its slope?

### Miscellaneous problems

45. Find the distance between the two points  $(2, -3)$  and  $(-5, 0)$ .
46. Consider the numbers  $\frac{2056}{2055}$  and  $\frac{2057}{2056}$ . Which of the following is true?
- (a)  $\frac{2056}{2055} < \frac{2057}{2056}$
- (b)  $\frac{2057}{2056} < \frac{2056}{2055}$
- (c)  $\frac{2056}{2055} = \frac{2057}{2056}$
- (d) It is impossible to tell without a calculator which of these numbers is larger.
- (e) None of the above
47. What is the range of the function  $f(x) = 10(x+7)^2 + 6$ ?
48. If  $z = (4x + 5y)^2$  then  $z =$
49. Find the domain of the function  $f(x) = \sqrt{\frac{2x+5}{3x-2}}$ .

## 2 Answers

### Values of trig. functions at reference angles

1. Find  $\cos(-\frac{1}{6}\pi)$ .
- Answer:
- $\frac{1}{2}\sqrt{3}$

2. Find  $\csc(\frac{5}{6}\pi)$ .

Answer:  
2

3. Find  $\sin(\frac{2}{3}\pi)$ .

Answer:  
 $\frac{1}{2}\sqrt{3}$

4. Find  $\tan(\frac{11}{6}\pi)$ .

Answer:  
 $-\frac{1}{3}\sqrt{3}$

5. Find  $\sec(\frac{7}{6}\pi)$ .

Answer:  
 $-\frac{2}{3}\sqrt{3}$

6. Find  $\cot(\frac{4}{3}\pi)$ .

Answer:  
 $\frac{1}{3}\sqrt{3}$

### Trig identities

7. Give the formula for  $\cos(x-y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .

Answer:  
 $\cos x \cos y + \sin x \sin y$  (Trig identities.)

8. Give the formula for  $\cos(2x)$  in terms of  $\cos x$  and  $\sin x$ .

Answer:  
 $\cos^2 x - \sin^2 x$  (Trig identities.)

9. Give the formula for  $\cos(x+y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .

Answer:  
 $\cos x \cos y - \sin x \sin y$  (Trig identities.)

10. Give the formula for  $\sin(x-y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .

Answer:  
 $\sin x \cos y - \cos x \sin y$  (Trig identities.)

11. Give the formula for  $\sin(x+y)$  in terms of  $\cos x$ ,  $\sin x$ ,  $\sin y$ , and  $\cos y$ .

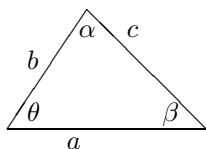
Answer:  
 $\sin x \cos y + \cos x \sin y$  (Trig identities.)

12. Give the formula for  $\sin(2x)$  in terms of  $\cos x$ ,  $\sin x$ .

Answer:  
 $2 \cos x \sin y$  (Trig identities.)

### Law of cosines and sines

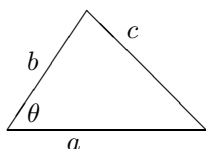
13. In the picture,  $\theta = \frac{1}{4}\pi$ ,  $\alpha = \frac{2}{3}\pi$  and  $c = 2$ . Find  $a$ .



Answer:

$$\sqrt{3}\sqrt{2}.$$

14. In the picture,  $a = 5$ ,  $b = 3$ , and  $\theta = \frac{1}{6}\pi$ . Find  $c$ .



Answer:

$$\sqrt{(34 - 15\sqrt{3})}.$$

### Inverses

15. Let  $f(x) = \frac{3x-7}{11x+5}$ . Find  $f^{-1}(x)$ .

Answer:

To find the inverse, we switch  $x$  and  $y$  in  $y = \frac{3x-7}{11x+5}$  and then solve for  $y$ . Thus we must solve for  $y$  in the equation,  $x = \frac{3y-7}{11y+5}$ . To solve this we multiply both sides by  $11y+5$  and this gives  $3y-7 = 11xy+5x$ . Now we collect the terms having a  $y$  on one side and the other terms on the other side. After factoring out  $y$ , this yields  $y(3-11x) = 5x+7$ . Therefore,  $y = f^{-1}(x) = \frac{5x+7}{3-11x}$ .

16. Suppose  $rx^2 + sx + t = 0$  where  $r \neq 0$ . Then  $x =$

Answer:

This is just the quadratic formula. The answer is  $\frac{-s \pm \sqrt{s^2 - 4rt}}{2r}$ .

17. Find all solutions to  $x^4 - 11x^2 + 30 = 0$ .

Answer:

$x = \pm\sqrt{6}$  and  $x = \pm\sqrt{5}$ . (Solving quadratic equations and factoring.)

18. Find all solutions to  $x^2 + 5x + 6 = 0$

Answer:

$$x = -2 \text{ and } x = -3$$

19. Suppose  $-2x + y = -4$  and  $x - 4y = -4$ . Find  $x - y$ .

Answer:

We multiply the second equation by  $-2$  and then subtract from the first equation. This yields  $-7y = -12$ , Solution is :  $\{y = \frac{12}{7}\}$ . Now we know what  $y$  is, we plug in to the second equation to find  $x$ . Thus  $x - \frac{48}{7} = -4$ , Solution is :  $\{x = \frac{20}{7}\}$ . Finally we find  $x - y = \frac{8}{7}$ .

20. For what value of  $k$  does the system of equations,  $2x + y = 1$ ,  $12x + ky = 12$  have no solution?

Answer:

$$6$$

### Word problems

21. A water tank is initially  $\frac{1}{4}$  full. After adding 14 gallons of water, it is  $\frac{7}{8}$  full. What is the capacity of the tank in gallons?

Answer:

Initially there are  $\frac{1}{4}x$  gallons in the tank where  $x$  is the capacity of the tank. When we add 14 this gives us  $\frac{1}{4}x + 14$  which is given to equal  $\frac{7}{8}x$  gallons. Thus we need to solve the equation,  $\frac{1}{4}x + 14 = \frac{7}{8}x$  for  $x$ . Solution is :  $\{x = \frac{112}{5}\}$ .

22. The perimeter of a rectangle is 9 times its width. If the length of the rectangle is 49, find its width.

Answer:

Let the width of the rectangle be  $x$  and let its length be  $y$ . Then  $2x + 2y = 9x$ . Also, we know that  $y = 49$ . Substituting this in, we find  $2x + 98 = 9x$ , Solution is :  $\{x = 14\}$ .

### Solving inequalities

23. Solve  $x(x+5)(x-4) < 0$  for  $x$ .

Answer:

$(-\infty, -5) \cup (0, 4)$ . (absolute values and solving inequalities.)

24. Solve  $|5x - 25| < 20$  for  $x$ .

Answer:

$(1, 9)$ . (absolute values and solving inequalities.)

25. Find the solution to the inequality,  $|4x - 2| < 6$

Answer:

The inequality holds if and only if  $-6 < 4x - 2 < 6$ . Therefore, we need to have  $-4 < 4x < 8$ . Since  $4 > 0$  we can divide by it and get  $-1 < x < 2$ .

### Solving Equations

26. Let  $y = \frac{7x+5}{3x-7}$ . Solve for  $x$ .

Answer:

To solve this we multiply both sides by  $3x - 7$  and this gives  $7x + 5 = 3xy - 7y$ . Now we collect the terms having an  $x$  on one side and the other terms on the other side. After factoring out  $x$ , this yields  $x(7 - 3y) = -7y - 5$ . Therefore,  $x = \frac{-7y-5}{7-3y}$ .

27. Solve for  $x$  in the equation  $10(1 + 17x)^{1/2} + 85x(1 + 17x)^{-1/2} = 0$ .

Answer:

We multiply both sides by  $(1 + 17x)^{1/2}$  and this gives  $10(1 + 17x) + 85x = 0$ . Now we just solve this for  $x$ .  $x = -\frac{2}{51}$ .

28. Find all positive solutions to  $\sqrt{(x^2 + 16)} = 6$ .

Answer:

We square both sides and get  $x^2 = 20$  and so  $x = \sqrt{20}$ .

### Simplifying expressions

29. Simplify  $(3^4)^{\frac{-1}{4}}$ .

Answer:

$$\frac{1}{3}$$

30. Simplify  $\frac{2xy^3}{2^{-2}xy^2}$ .

Answer:

This involves using the rules of exponents. The answer is  $8y$ .

Simplifying

31. Simplify  $\frac{1}{x^2+4} + \frac{2}{x}$ .

Answer:

We have to add these. Thus we need to put the two fractions over the same common denominator.

$$\frac{x}{(x^2+4)x} + \frac{2(x^2+4)}{x(x^2+4)} = \frac{2x^2+x+8}{(x^2+4)x}$$

32. Simplify  $\frac{1}{x^2-4} + \frac{2}{3x-6}$ .

Answer:

$$\frac{1}{3} \frac{2x+7}{(x-2)(x+2)}$$

33. Simplify  $|\sqrt{7} - 7| - |-5|$ .

Answer:

$-\sqrt{7} + 2$  (exponents and radicals and absolute values.)

34. Simplify  $|\frac{1}{8} - \frac{1}{4}\sqrt{2}| - |-4|$ .

Answer:

$\frac{1}{4}\sqrt{2} - \frac{33}{8}$  (exponents and radicals and absolute values.)

Simplify difference quotient

35. Suppose  $f(x) = x^2 + 6x$ . Simplify  $\frac{f(x+h)-f(x)}{h}$ .

Answer:

$f(x+h) - f(x) = (x+h)^2 + 6(x+h) - [x^2 + 6x]$  which upon multiplying and cancelling is  $2xh + h^2 + 6h$ . It follows that if we divide by  $h$  we get  $2x + 6 + h$ .

### Factoring of polynomials

36. Factor the polynomial  $x^2 + 13x + 42$ .

Answer:

$$(x+7)(x+6)$$

37. Factor the polynomial  $6x^2 + 43x + 42$ .

Answer:

$$(6x+7)(x+6)$$

### Composition Of Functions

38. Suppose  $f(-2) = 3$ ,  $f(3) = -2$ , and  $f(1) = -1$  while  $g(3) = 1$ ,  $g(-2) = 2$ , and  $g(1) = 1$ . Find  $f(g(-2+3))$ .

Answer:

$$-1$$

39. Let  $f(x) = -3x^2 + 2x$  and let  $g(x) = -3x^2$ . Find  $f(g(1))$ .

Answer:

We see  $g(1) = -3$  and so  $f(g(1)) = f(-3) = -33$ .

### Lines

40. A line having the equation  $y = mx + b$  passes through the points  $(-4, -7)$  and  $(9, 1)$ . Find  $m + b$ .

Answer:

The slope of the line is  $\frac{8}{13}$  and so the equation of the line is  $y + 7 = (\frac{8}{13})(x + 4)$ . We need to find the  $y$  intercept. This is obtained when  $x = 0$ . Thus  $y + 7 = (\frac{8}{13})(4)$ , Solution is:  $\{b = -\frac{59}{13}\}$ . Now  $m + b = \frac{8}{13} - \frac{59}{13} = -\frac{51}{13}$ .

41. Find the equation of the line through the points  $(-4, 3)$ ,  $(5, 4)$ .

Answer:

$$y = \frac{1}{9}x + \frac{31}{9}$$
 (lines and distances.)

42. Find the equation of the line perpendicular to  $-18x + 6y = 6$  through  $(0, 6)$ .

Answer:

$$y = -\frac{1}{3}x + 6 \text{ (lines and distances.)}$$

43. Find the equation of the line parallel to  $-3x + y = 3$  and passing through  $(0, 8)$ .

Answer:

$$y = 3x + 8 \text{ (lines and distances.)}$$

44. A line has the equation  $6x = 5y + 10$ . What is its slope?

Answer:

We can put it in slope intercept form as follows.  $y = \frac{6}{5}x - 2$ . Therefore, the slope is  $\frac{6}{5}$ .

### Miscellaneous problems

45. Find the distance between the two points  $(2, -3)$  and  $(-5, 0)$ .

Answer:

$$\sqrt{(7)^2 + (-3)^2} = \sqrt{58}$$

46. Consider the numbers  $\frac{2056}{2055}$  and  $\frac{2057}{2056}$ . Which of the following is true?

- (a)  $\frac{2056}{2055} < \frac{2057}{2056}$
- (b)  $\frac{2057}{2056} < \frac{2056}{2055}$
- (c)  $\frac{2056}{2055} = \frac{2057}{2056}$
- (d) It is impossible to tell without a calculator which of these numbers is larger.
- (e) None of the above

Answer:

$$\frac{2056}{2055} = 1 + \frac{1}{2055} > 1 + \frac{1}{2056} = \frac{2057}{2056}.$$

47. What is the range of the function  $f(x) = 10(x + 7)^2 + 6$ ?

Answer:

All real numbers  $\geq 6$

48. If  $z = (4x + 5y)^2$  then  $z =$

Answer:

$$16x^2 + 40xy + 25y^2$$

49. Find the domain of the function  $f(x) = \sqrt{\frac{2x+5}{3x-2}}$

Answer:

$$\left(\frac{2}{3}, \infty\right) \cup \left(-\infty, -\frac{5}{2}\right]$$