Math 341 Exam 2 Preparation Sheet
Supplement: Answers

1. (a) True, because $\sum 1/n^{3/6}$ and $\sum 1/n^{5/6}$ both diverge, and $\sum 1/n^{3/6+5/6}$ converges.
1. (b) True. Proof by contradiciton: suppose $\sum a_n$ converges conditionally and $\sum n^2 a_n$ converges. For the latter we have $\lim n^2 a_n = 0$, so for $\epsilon = 1$ there is $N \in \mathbb{N}$ such that $|n^2 a_n| < 1$ for all $n \geq N$. This says that $|a_n| \leq 1/n^2$, so by the comparison test, the series $\sum |a_n|$ converges. By the Absolute Convergence Test the series $\sum a_n$ converges absolutely, contradiction the conditional convergence of $\sum a_n$.
2. (a) True, because the isolated point does not contain an open interval.
2. (b) True, every singleton set $\{x\}$ is closed (the complement $(-\infty, x) \cup (x, \infty)$ is open), and a finite union of closed sets is closed.
2. (c) False, counterexample $O = (-\infty, \sqrt{2}) \cup (\sqrt{2}, \infty)$.
3. (a) False, counterexample $A = [0, 1] \cup \{2\}$ where $2 = \sup A$ but 2 is not a limit point but an isolated point of A.
3. (b) True, because for each point a in A is a positive distance away from its nearest neighbours, and so we can find $\epsilon > 0$ such that $V_\epsilon(a) \cap A = \{a\}$.
4. (a) True, because $\overline{A}$ is closed, so its complement is open.
4. (b) True, because $\overline{A}$ is the union of A with its limit points, and a set is closed when it contains its limit points.
5. (a) True, because the intersection is a subset of A_1 and so bounded, and the intersection of closed sets is closed.
5. (b) False, counterexample $A_n = (-\infty, 1 + 1/n]$ where the intersection is $(\infty, 1]$ which is closed but not bounded.
6. (a) True, it is a theorem in the book that every perfect set is uncountable.
6. (b) True, because $C \cap [0, 1/2]$ is nonempty, closed, and has no isolated points.
6. (c) False, because $C \cap \mathbb{Q}$ is countable, and hence can not be perfect.
7. (a) True, because in between any $x, y \in C$ with $x < y$ there is $z \notin C$ such that $x < z < y$.
7. (b) True, this is a theorem in the book.
8. (a) True, because for $\epsilon > 0$ we choose $\delta = \min\{1, \epsilon/4\}$ which gives $|x^2 + x - (1+1)| < \epsilon$.
8. (b) False, because for $x_n = \exp(-n\pi/2)$ we have $\sin(\ln(x_n))$ oscillates between -1 and 1 as $n \to \infty$.
9. (a) False, because for $x \in C$, there are sequences (x_n) in C and $(y_n) \in [0, 1] \setminus C$ such that $g(x_n) = 1$ and $g(y_n) = 0$.
9. (b) True, this is a theorem in the book.