Math 113
Exam 3 PART A
Winter 2013

RED
TWO PART EXAM
DO NOT WRITE ON PART A

Instructions for Part A:

- Calculators are not allowed.
- Bubble your answer to the questions on the provided scantron. Use a #2 pencil.
- Please do not discuss the exam with other students until after the last day to take the exam.

MULTIPLE CHOICE (3 points each)

1. Given one sequence $\{a_n\} = \{a_1, a_2, a_3, \dots\}$, define a second sequence $\{s_n\}$ recursively by $s_1 = a_1$ and $s_{n+1} = s_n + a_{n+1}$. Then the sequence $\{s_n\}$ converges if and only if...

   (a) the sequence $\{a_n\}$ converges
   (b) the sequence $\{a_n\}$ converges to 0
   (c) the sequence $\{a_n\}$ is decreasing
   (d) the sequence $\{a_n\}$ diverges
   (e) the series $\sum_{n=1}^{\infty} a_n$ diverges
   (f) the series $\sum_{n=1}^{\infty} a_n$ converges

2. Let $a_1 = 1$ and $a_{n+1} = 2 + \frac{3}{a_n}$. Find a_3.

   (a) 3
   (b) $\frac{13}{5}$
   (c) $\frac{11}{5}$
   (d) 2
   (e) $\frac{9}{5}$
   (f) none of the above

3. If convergent, find the sum of the series: $\sum_{n=1}^{\infty} \frac{2}{5^n}$

   (a) 2
   (b) $\frac{1}{2}$
   (c) $\frac{2}{3}$
   (d) 1
   (e) $\frac{2}{5}$
   (f) divergent

4. If convergent, find the sum of the series: $\sum_{n=2}^{\infty} \left(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n+2}} \right)$

   (a) $\frac{\sqrt{3}}{3} - \frac{1}{2}$
   (b) $\sqrt{2}$
   (c) $\sqrt{3}$
   (d) $\frac{\sqrt{3}}{3}$
   (e) $\frac{\sqrt{2}}{2}$
   (f) divergent

CONTINUE TO THE NEXT PAGE

5. Choose the test which concludes that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

(a) alternating series test
(b) ratio test
(c) root test
(d) integral test
(e) divergence test
(f) love test

6. Write the first three nonzero terms of a power series representation for the function: $\frac{2x}{4x+1}$

(a) $2x + 16x^2 - 32x^3 \cdots$
(b) $2x - 8x^2 + 16x^3 \cdots$
(c) $2x - 8x^2 + 32x^3 \cdots$
(d) $2x + 8x^2 + 16x^3 \cdots$
(e) $2x - 16x^2 + 64x^3 \cdots$
(f) $2x + 16x^2 + 64x^3 \cdots$

7. Find the radius of convergence for the power series: $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{4n}$

(a) 1
(b) $\frac{1}{3}$
(c) 0
(d) $\frac{1}{\sqrt{2}}$
(e) $\frac{1}{4}$
(f) ∞

8. Using the geometric series expansion of $\frac{1}{1-x}$, find a power series representation for $\frac{3}{(1-x)^2}$.

(a) $\sum_{n=0}^{\infty} 3nx^{n+1}$
(b) $\sum_{n=0}^{\infty} n3^n x^{n+1}$
(c) $\sum_{n=0}^{\infty} 3nx^n$
(d) $\sum_{n=0}^{\infty} 3^n x^{n+1}$
(e) $\sum_{n=0}^{\infty} 3(n+1)x^n$
(f) $\sum_{n=0}^{\infty} (n+1)3^n x^n$

9. Suppose the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. Which of the following must be true?

(a) $\sum_{n=1}^{\infty} (-1)^n a_n$ is conditionally convergent
(b) $\sum_{n=1}^{\infty} a_n$ has a positive sum
(c) $\{a_n\}_{n=1}^{\infty}$ alternates in sign
(d) $\sum_{n=1}^{\infty} (-a_n)$ is absolutely convergent
(e) $\{a_n\}_{n=1}^{\infty}$ has positive terms only
(f) none of the other options must be true

10. If any, which of the following series is convergent?

(a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$
(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$
(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
(e) $\sum_{n=1}^{\infty} \frac{1}{n^{-1}}$
(f) They are all divergent

END OF PART A