Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 113 (Calculus II)
Exam 2 RED

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Calculators are not allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the page the problem is on and indicate this fact.
- Please do not talk about the test with other students until after the last day to take the exam.

For Instructor use only.

#	Possible	Earned
MC	36	
10	8	
11	8	
12	6	
13	6	
Sub	64	

#	Possible	Earned
14	6	
15	12	
16	6	
17	6	
18	6	
Sub	36	
Total	100	

Optional Problem (9 possible)	

Part I: Multiple Choice *Mark the correct answer on the bubble sheet provided.*

1. The integral $\int_{-\infty}^{0} e^{-2x}\,dx$ equals:

   a) -1 b) $-\frac{1}{2}$ c) 0

   d) $\frac{1}{2}$ e) 1 f) e

   g) The integral does not converge. h) None of the above

2. For the following probability density function f given below, find $P(\frac{\pi}{3} \leq X \leq \frac{\pi}{2})$.

$$f(x) = \begin{cases} \frac{\sin(x)}{2} & \text{if } 0 \leq x \leq \pi \\ 0 & \text{otherwise.} \end{cases}$$

   a) 1 b) $\frac{1}{2}$ c) $\frac{1}{4}$

   d) $\frac{\sqrt{3}}{4}$ e) $\frac{\sqrt{2}}{4}$ f) None of these

3. Which if the following approximations will be the closest to the actual value of the integral $\int_{0}^{3\pi} \sin(x)\,dx$? (Hint: Think of the graph for the function).

   a) T_3 b) M_3 c) R_3

   d) L_3 e) Each of these estimates will give the same approximation.

Continue to Next Page

4. If the function f is defined by

$$f(x) = \begin{cases} k(4x - x^2) & \text{if } 0 \le x \le 4 \\ 0 & \text{if } x < 0 \text{ or } x > 4, \end{cases}$$

for what value of k is $f(x)$ a probability density function?

a) $\dfrac{3}{80}$ b) $\dfrac{3}{32}$

c) $\dfrac{1}{4}$ d) $\dfrac{1}{11}$

e) $\dfrac{1}{30}$ f) $\dfrac{3}{112}$

g) None of these

5. The integral $\displaystyle\int_1^4 \frac{1}{x-2}\,dx$ equals:

a) $\ln(2)$ b) 0

c) $\ln(4)$ d) $\ln(3)$

e) $\dfrac{3}{4}$ f) The integral does not converge.

g) None of these

6. The integral $\displaystyle\int_{-1}^0 \frac{x^2+2}{x+2}\,dx$ equals:

a) $6\ln(2)$ b) $6\ln(2) + \dfrac{3}{4}$

c) $6\ln(2) - \dfrac{3}{4}$ d) $6\ln(2) - \dfrac{5}{2}$

e) The integral does not converge. f) None of these

7. Use the trapezoidal rule with $n = 2$ to approximate the integral $\displaystyle\int_{-1}^3 x^4\,dx$

a) 84 b) 83

c) 42 d) 21

e) 242/5 f) None of these

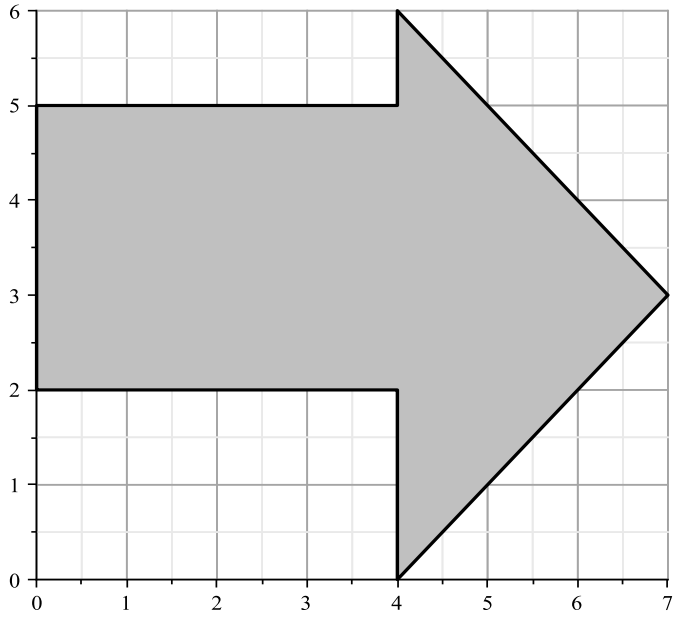
Continue to Next Page

8. Simpson's rule with n subdivisions, where n is even, is used to approximate the integral $\int_0^{\pi/2} \sin(2x)\,dx$. If E_S is the error in using Simpson's Rule, then which is the correct theoretical upper bound on the error for this approximation?

a) $|E_S| \leq \dfrac{2^4\pi^4}{180n^3}$ b) $|E_S| \leq \dfrac{\pi^3}{24n^2}$ c) $|E_S| \leq \dfrac{\pi^3}{48n^2}$

d) $|E_S| \leq \dfrac{\pi^5}{360n^4}$ e) $|E_S| \leq \dfrac{2^5\pi^4}{12n^3}$ f) $|E_S| \leq \dfrac{\pi^5}{96n^4}$

9. Find the y coordinate of the centroid of the shaded region in the following figure:



a) 3 b) 4

c) $\dfrac{7}{2}$ d) $\dfrac{16}{5}$

e) $\dfrac{23}{7}$ f) None of these

Continue to Next Page

Part II: Short Answer. *Evaluate or give the best response in the blank provided. Work will not be graded in the section. Only the answer will be graded. Questions are worth 2 points a piece.*

10. (8 points)

(a) Set up, but do not evaluate, an integral to find the arclength of $f(x) = e^{2x}$ from $x = -2$ to $x = 3$.

_____

(b) Give the formula for finding the surface area of revolution of a function $f(x)$ from $x = a$ to $x = b$ rotated around the x-axis.

_____

(c) $\displaystyle\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx$ _____

(d) $\displaystyle\int_1^2 \frac{1}{\sqrt{x-1}}\,dx$ _____

11. (8 points)

(a) Calculate the hydrostatic force on the top of a cube shaped box with side length 0.5 feet. The top of the box is 4 feet below the surface of the water (use 62.5 lbs per cubic foot as the weight density of water).

_____

(b) Calculate the hydrostatic force on one of the vertical sides of the box in problem 11(a).

_____

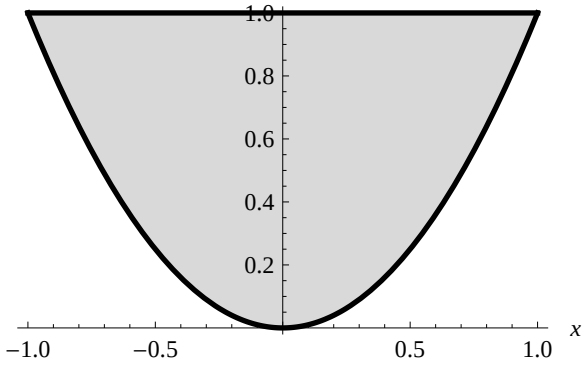
(c) Set up (but do not solve) the partial fraction decomposition for $\dfrac{3x+1}{(x^2+2)^2(x+2)^2}$.

_____

(d) The indefinite integral $\displaystyle\int e^{x+e^x}\,dx$ equals _____

Part III: Free Response *Show all work.*

12. (6 points) Calculate the hydrostatic force on the end of a trough with the end shaped like a parabola with the equation $y = x^2$. The trough is 1 foot high and is filled to the top with liquid. Use 60 lbs/ft^3 as the weight density of the liquid. The end of the trough is pictured below.



13. (6 points) Find the mean of the following distribution:

$$f(x) = \begin{cases} \frac{2}{\sqrt{\pi}} e^{-x^2} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

14. (6 points) Find the arc length of the curve $y = \ln(\sec(x))$ for $0 \leq x \leq \frac{\pi}{4}$.

Continue to Next Page

15. (12 points) Evaluate the following integrals (4 points a piece):

(a) $\displaystyle\int \frac{1}{(x-1)(x+1)}\,dx$

(b) $\displaystyle\int \cos^3(x)\sin^3(x)\,dx$

(c) $\displaystyle\int \frac{1}{x^3-3x^2-10x}\,dx$

16. (6 points) Find the area of the surface of revolution obtained by rotating the curve $y = \dfrac{x^3}{3}$ about the x-axis for $0 \leq x \leq 2$.

17. (6 points) Find the centroid of the region bounded by the curves $y = x^2$ and $y = 2x$.

Continue to Next Page

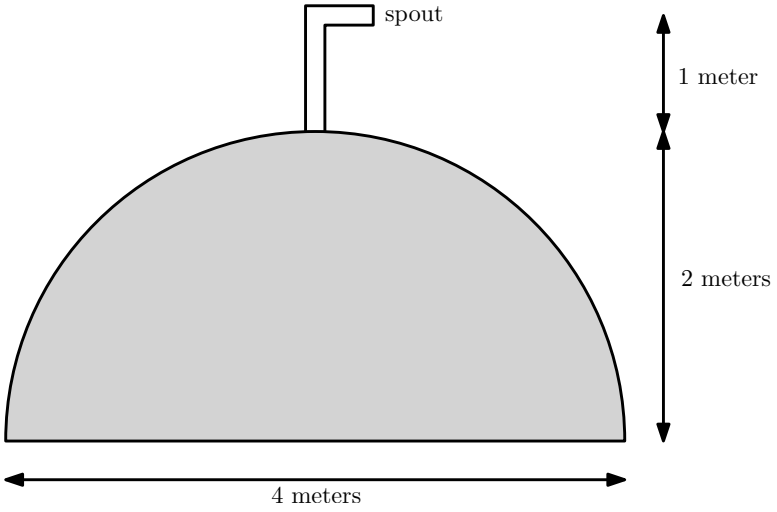
18. (6 points) We know that $\int_1^2 \frac{1}{x}\,dx = \ln(2)$. We can use the area under the curve $y = \frac{1}{x}$ to estimate the value of $\ln(2)$. Use the midpoint approximation on $y = \frac{1}{x}$ with 2 subintervals to estimate the value of $\ln(2)$.

Continue to Next Page

Optional Question: *On the first midterm exam, question 15 mistakenly did not specify the weight density of the rope. If you are not satisfied with the score you received on that question, you may answer the following* ***optional*** *question on the topic of work. The original score on midterm exam 1 problem 15 will be replaced by the higher of the two scores.*

An underground tank is shaped like the upper half of a sphere. The top of the tank is 1 meter below the ground, and the radius of the tank is 2 meters, as indicated in the figure below. Initially, the tank is full of liquid. Assuming the weight density of the liquid is ρ, calculate the work W required to empty the tank by pumping the liquid to the top of the spout.

[Note: You should set up an integral for W and then evaluate the integral. The answer will depend on the physical constant ρ.]



END OF EXAM