MATH 113 – EXAM 3 – PART 1

NOVEMBER 13–16, 2012

Instructions:

1. This is the first of two parts.
2. Record the answers to the multiple choice questions on the bubble sheet.
3. Notes, books, and calculators are not allowed.
4. Please do not talk about the test with other students until after the last day of the exam.

DO NOT WRITE ON THIS PART OF THE EXAM

1. Which of the following represents the general term of the sequence:

$$\frac{2}{1}, -\frac{6}{3}, \frac{12}{5}, -\frac{20}{7}, \frac{30}{9}, -\frac{42}{11}, \frac{56}{13}, \ldots$$

A) $(-1)^{n+1}\dfrac{n(n+1)}{n!}$ B) $(-1)^{n+1}\dfrac{n+1}{n}$ C) $(-1)^{n+1}\dfrac{n^2+1}{n}$

D) $(-1)^{n+1}\dfrac{n^2+1}{2n-1}$ E) $(-1)^{n+1}\dfrac{n(n+1)}{2n-1}$ F) $(-1)^{n+1}\dfrac{n+1}{n!}$

2. Consider the sequence:

$$a_n = \frac{6n-1}{n+3}$$

The sequence is

A) decreasing and not bounded.
B) increasing and bounded.
C) bounded, and neither increasing nor decreasing.
D) decreasing and bounded.
E) not bounded, and neither increasing nor decreasing.
F) increasing and not bounded.

3. Consider the series

$$\sum_{k=1}^{\infty} (-1)^{k+1}\frac{e^k}{k!}.$$

The series is

A) absolutely convergent.
B) absolutely convergent but conditionally divergent.
C) divergent.
D) conditionally convergent but absolutely divergent.
E) conditionally convergent.
F) Convergence of this series cannot be determined.

4. Choose from among the options the smallest choice of n for which the error is no more than 0.001 for the series:

$$\sum_{k=1}^{\infty} (-1)^{k+1}\frac{1}{k^3}$$

A) 3 B) 2 C) 11
D) 10 E) 32 F) 5

Do no write on this portion of the test.

5. Which tests should we use to determine the convergence of the three series

$$\sum_{k=1}^{\infty} \frac{1}{k^2+3} \qquad \sum_{k=1}^{\infty} \frac{7^k}{(k+2)!} \qquad \sum_{k=1}^{\infty} \frac{k}{(k+1)^{k^2}} \quad ?$$

List the correct tests in the same order as the series.

A) Integral, Ratio, and Root
B) Integral, p-Series, and Root
C) Divergence, Ratio, and Geometric
D) Divergence, Root, and p-Series
E) Root, Ratio, and p-Series
F) Integral, Ratio, and Geometric

6. Estimate the upper bound of the error in summing the first 16 terms to calculate

$$\sum_{k=1}^{\infty} \frac{\sqrt{n}}{(n^{3/2}+2)^2}$$

A) 1/100
B) 1/50
C) 3/100
D) 2/99
E) 1/33
F) 1/99

7. Which of the following statement(s) is/are false

(i) If $\sum a_n$ converges, then $\sum \frac{1}{a_n}$ diverges

(ii) Given a convergent series $\sum a_n$ of positive terms, the series $\sum \sqrt{a_n}$ may not be convergent.

(iii) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n^2+3n+1}}$ converges because $\lim_{n\to\infty} \frac{1}{\sqrt{2n^2+3n+1}} = 0$.

(iv) If $\int_1^{\infty} f(x)\,dx = 5$, then the sum of the series $\sum_{n=1}^{\infty} f(n)$ is 5.

A) (ii) only
B) (iv) only
C) (i) and (ii)
D) (ii) and (iii)
E) (iii) and (iv)
F) (ii), (iii), and (iv)

8. The series $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$ is convergent. Find its sum.

A) 2/4
B) 3/2
C) 1/3
D) 2
E) 7/3
F) 7/4

Do no write on this portion of the test.

9. Consider the converging sequence $\{a_n\}$ generated by the recurrence relation

$$a_1 = 5 \qquad a_{n+1} = (3a_n + 2)/4 \quad \text{for } n = 1, 2, 3, \ldots$$

Find the limit of the sequence.

A) 7 B) -1 C) 0
D) 2 E) 5 F) -3

10. Find values of x for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{8^n}$ converges and find the sum for those values of x.

A) $(-1, 1)$, sum is $\dfrac{x}{8-x}$ B) $[-1, 1]$, sum is $\dfrac{8}{x-8}$

C) $(-8, 8)$, sum is $\dfrac{8}{x-8}$ D) $(-8, 8)$, sum is $\dfrac{x}{8-x}$

E) $(0, 1)$, sum is $\dfrac{8}{x-8}$ F) $[0, 8]$, sum is $\dfrac{x}{8-x}$

11. Given the two series

$$A = \sum_{n=1}^{\infty} \frac{\sin^2(5n)}{n^{10}\sqrt{n}} \qquad \text{and} \qquad B = \sum_{n=1}^{\infty} 8\cos\left(\frac{1}{7n}\right),$$

determine whether each series is convergent or divergent.

(a) By the Integral Test, A is convergent and B is divergent.
(b) By the Comparison Test, both A and B are divergent.
(c) A is convergent by the Comparison Test. B is divergent by Divergence Test.
(d) A is convergent by the Comparison Test. B is convergent by the Integral Test.
(e) By the Limit Comparison Test, both A and B are convergent.
(f) A is divergent by the Integral test. B is convergent by the Limit Comparison Test.

Do no write on this portion of the test.