1. If a system of linear equations has a solution, we say it is ———————————-.

2. Which of the following is a solution to the system of equations

$$\begin{aligned} x_1 + x_2 - x_3 - x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 8 \\ 2x_1 - x_2 + x_3 &= 0 \\ x_2 - x_4 &= 0 \end{aligned}$$

   (a) $(1, 1, 3, 3)$
   (b) $(1, 3, 1, 3)$
   (c) $(3, 3, 1, 1)$
   (d) $(1, -1, 3, -3)$
   (e) $(1, -3, 1, -3)$
   (f) $(-1, 1, -3, 3)$

3. Consider the following augmented matrix:

$$\begin{bmatrix} 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & -1 & 1 & -1 \\ 4 & 5 & 0 & k & 2 \end{bmatrix}$$

   For which value of k is the system inconsistent?

   (a) 0
   (b) 1
   (c) 2
   (d) 4
   (e) 6
   (f) It is consistent for all values of k.
   (g) It is inconsistent for all values of k.

4. Find all solutions to the system of equations

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 1 \\ 2x_1 + x_3 + x_4 &= 0 \\ x_1 + 2x_2 + x_3 + 2x_4 &= 1 \\ x_1 + x_2 &= -1 \end{aligned}$$

5. Which of the following statements are not necessarily true for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$?

   (a) If $T(\mathbf{x}) = \mathbf{0}$, then $\mathbf{x} = \mathbf{0}$.

   (b) $T(\mathbf{0}) = \mathbf{0}$

   (c) $T(\alpha\mathbf{x}) = \alpha\mathbf{x}$

   (d) $T(\mathbf{x} + \mathbf{y}) = \mathbf{T}(\mathbf{x}) + \mathbf{T}(\mathbf{y})$

   (e) A vector rotation through angle θ is a linear transformation.

6. Every linear transformation can be represented by a _____.

7. Define $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$ by the following matrix

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ -1 & -3 \end{bmatrix}.$$

   Which of the following vectors is not in the range of T?

   (a) $\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

   (c) $\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$

   (d) $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

8. Define

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_3 \\ 2x_1 - x_2 + x_3 \\ x_1 - 2x_2 + x_3 \end{bmatrix}.$$

(a) State the matrix of T.

(b) Find all values $\mathbf{x}$ where $T(\mathbf{x}) = \mathbf{0}$.