Name:________________________________

Student ID:________________________________

Section:________________________________

Instructor:

Math 313 (Linear Algebra)
Exam 2 - Practice Exam

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Scientific calculators are allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the page the problem is on and indicate this fact.
- Please do not talk about the test with other students until after the last day to take the exam.

Part I: Multiple Choice Questions: *Mark **all** answers which are correct for each question.*

1. Let A be a 3×3 matrix such that the columns of A sum to the zero vector. Let $B = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

   Which of the following could be the product matrix AB? Select all that apply.

   a) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

   b) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

   c) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

2. Which of the following rules for determinants concerning $n \times n$ matrices A and B are correct. Select all that apply.

   a) $\det(AB) = \det(A)\det(B)$.

   b) $\det(A + B) = \det(A) + \det(B)$.

   c) $\det(AB) = \det(BA)$.

   d) $\det(A^T) = \det(A)$.

   e) Swapping two rows in A results in a change of the sign of the corresponding determinant.

   f) Subtracting column number 2 from column number 1 does not alter the value of the determinant.

3. Which of the following statements is true? Select all that apply.

   a) $\det(A^T B) = \det(B^T A)$.

   b) In a determinant of a 3×3-matrix A one may swap the first row and the first column without changing the value of the determinant.

   c) Adding to the first column of a determinant a linear combination of the remaining columns does not affect its value.

   d) A determinant is linear as a function of each of its vector arguments.

   e) If the determinant of a 3×3-matrix is equal to the product of its elements in the main diagonal then it is either an upper triangular or a lower triangular matrix.

   f) For a 4×4-matrix A one always has $\det(2A) = 16\det(A)$.

4. Which of the following are vector spaces? (Addition and scalar multiplication are defined normally.) Select all that apply.

   a) The set of all polynomials with even coefficients.

   b) The set of all polynomials with odd coefficients.

   c) The set of all vectors in $\mathbb{R}^3$ with the form $\begin{bmatrix} a+1 \\ a+b \\ a+c \end{bmatrix}$ where a, b, c are real numbers.

   d) The set of all vectors in $\mathbb{R}^3$ with the form $\begin{bmatrix} q+t \\ 2r-t \\ 3s+t \end{bmatrix}$ where q, r, s, t are real numbers.

   e) The set of all vectors in $\mathbb{R}^2$ of the form $\begin{bmatrix} x \\ y \end{bmatrix}$ where $y = x + 1$.

   f) The set of all real-valued functions f on $\mathbb{R}$ such that $f(1) = 1$.

5. Which of the following sets form a basis for $\mathbb{P}_2$? Select all that apply.

   a) $\{1, t, t^2\}$

   b) $\{t^2 + t - 2, -t + 2\}$

   c) $\{t^2 + 7, 2t - 3, t + 5, t^2 + t + 1\}$

   d) $\{2, t^2 + t, 2t^2 + 2t + 3\}$

   e) $\{1, t, t - 1\}$

   f) $\{t^2 + 2t + 3, 7t, t^2 + 1\}$

Part II: *Fill in the blank with the **best** possible answer.*

6. Let A and B be $n \times n$ matrices and express them in terms of their column vectors: $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix}$ and $B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_n \end{bmatrix}$. Then the products AB and BA can also be expressed in terms of their column vectors:

$$AB = \underline{\hspace{6cm}}$$

$$BA = \underline{\hspace{6cm}}.$$

7. Let A be a 2×2 matrix. If $\det(A) = -3$, then we know that A is ________________________.

8. Suppose that S is a region in the plane with area 15. Let $T(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix}$. Then the area of $T(S)$ is ________________.

Part III: *Justify your answer and show all work for full credit.*

9. Consider the matrix

$$A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{bmatrix}.$$

Show that the matrix A is invertible by finding the inverse of A using row reductions. Verify that the matrix that you find is the actual inverse of the matrix A.

10. Find the determinant of

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 1 & -1 \\ 2 & 1 & 1 & 0 & -3 \\ 0 & 1 & 0 & 2 & 1 \\ -2 & 2 & -1 & 1 & 0 \end{bmatrix}.$$

11. Suppose that x, y, and z satisfy

$$\begin{array}{rcrcrcr} 2x & - & 3y & + & 3z & = & 1 \\ 2x & - & y & + & z & = & 1 \\ x & + & 2y & + & z & = & -2 \end{array}.$$

Use Cramer's rule to find the value of y.

12. Let

$$A = \begin{bmatrix} 1 & -3 & 4 \\ 1 & -3 & 3 \\ 2 & -2 & 2 \end{bmatrix}.$$

Find the adjugate of A, and use it to compute A^{-1}.

13. Find the volume of the parallelepiped with one vertex at the origin, and adjacent vertices at $(1, 0, -2)$, $(1, 2, 4)$, and $(7, 1, 0)$.

14. Find bases for Nul A and Col A, where

$$A = \begin{bmatrix} 1 & 1 & 0 & -2 & 1 \\ 2 & 2 & 1 & -1 & 1 \\ -1 & 1 & 3 & 2 & -2 \end{bmatrix}.$$

15. Let H be the subspace of $\mathbb{P}_3$ made up of only those polynomials $\mathbf{p}(x)$ such that $\mathbf{p}(0) = 0$,

$$H = \{\mathbf{p}(x) \in \mathbb{P}_3 \mid \mathbf{p}(0) = 0\}.$$

Find a basis for H.

16. Let $M_{n\times n}$ be the vector space of all $n \times n$ matrices, and let $C \in M_{n\times n}$ be a fixed $n \times n$ matrix. Let H be the set of all $n \times n$ matrices that commute with A. Is H a subspace of $M_{n\times n}$? Justify your answer.

17. Let V be a vector space, with zero vector $\mathbf{0} \in V$. Using the vector space axioms to justify each step (you don't need to have them memorized), prove that $c \cdot \mathbf{0} = \mathbf{0}$ for any scalar $c \in \mathbb{R}$.

18. Let $T : V \to W$ be a linear transformation from a vector space V into a vector space W. Prove that the range of T is a subspace of W.

19. The column space of an $m \times m$ matrix A is all of $\mathbb{R}^m$ if and only if the equation $A\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b}$ in $\mathbb{R}^m$.

20. Prove the following statement if it is true, or provide a counterexample if it is false. Given a 3×3 matrix with columns $A = \begin{bmatrix} \mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} \end{bmatrix}$, then $\det(A) = \det\left(\begin{bmatrix} \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_1} \end{bmatrix}\right)$.

END OF EXAM