



NS



001

Math 313
Exam 1
May 12,13, 2015

Name: Key
Section: _____
Instructor: _____

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Instructions

- I) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 4 points each.
- III) Multiple choice questions that have more than one correct answer will be marked with a ♣. All other questions have only one correct answer.
- IV) For questions which require a written answer, show all your work in the space provided and justify your answer.
- V) Simplify your answers.
- VI) Scientific calculators are allowed.
- VII) No books or notes are allowed.
- VIII) There is no time limit on this exam.



Part I: Multiple Choice Questions: Questions marked with a ♣ have more than one correct answer. Mark **all** correct answers. The other questions have one right answer. (4 points each)

1 For which value of the constant b does the system $x_1 + 3x_2 = 1$, $x_1 + bx_2 = -1$ have no solution?

$b = -1$

$b = 0$

$b = 1$

$b = -2$

$b = 3$

$b = 2$

2 A row echelon form of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$ is

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

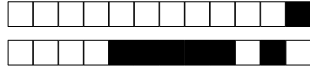
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$



3 For an $m \times n$ matrix A and a vector \mathbf{b} in \mathbb{R}^m , which of the following are logically equivalent to the statement: the columns of A span \mathbb{R}^m ?

- (i) Each \mathbf{b} in \mathbb{R}^m is a linear combination of the columns of A .
- (ii) Each row of A contains a pivot position.
- (iii) The value of m is larger than the value of n .

- | | |
|---|---|
| <input type="checkbox"/> only (i) | <input checked="" type="checkbox"/> only (i) and (ii) |
| <input type="checkbox"/> only (iii) | <input type="checkbox"/> only (ii) |
| <input type="checkbox"/> only (i) and (iii) | <input type="checkbox"/> none of them. |
| <input type="checkbox"/> all of them | <input type="checkbox"/> only (ii) and (iii) |

4 Which of the following three statements about the matrix equation $A\vec{x} = \vec{b}$ are true?

- (i) If A is a 4×2 matrix, then for every \vec{b} in \mathbb{R}^4 , the matrix equation $A\vec{x} = \vec{b}$ has a least one solution.
- (ii) If A is a 2×4 matrix, then for every \vec{b} in \mathbb{R}^2 , the matrix equation $A\vec{x} = \vec{b}$ can never have a unique solution.
- (iii) If A is a 3×3 , then for every \vec{b} in \mathbb{R}^3 , the matrix equation $A\vec{x} = \vec{b}$ never has a solution.

- | | |
|--|---|
| <input type="checkbox"/> only (i) and (ii) | <input checked="" type="checkbox"/> only (ii) |
| <input type="checkbox"/> none of them. | <input type="checkbox"/> only (i) and (iii) |
| <input type="checkbox"/> only (i) | <input type="checkbox"/> all of them |
| <input type="checkbox"/> only (iii) | <input type="checkbox"/> only (ii) and (iii) |



5 ♣ Which of the following statements are true? (Assume any multiplication and additions are defined). Mark all that apply.

- If every row of A is a pivot row of A , then $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
- A homogeneous system $A\mathbf{x} = \mathbf{0}$ is always consistent.
- The system $A\mathbf{x} = \mathbf{b}$ can be inconsistent even though $A\mathbf{x} = \mathbf{0}$ is consistent.
- The system $A\mathbf{x} = \mathbf{b}$ can have a unique solution even though $A\mathbf{x} = \mathbf{0}$ does not.
- The system $A\mathbf{x} = \mathbf{0}$ can have a unique solution even though $A\mathbf{x} = \mathbf{b}$ does not.

6 ♣ Which of the following sets are linearly independent? Mark all that apply.

- $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$
- $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 2 & -6 \end{bmatrix}$$



7 Which of the three vectors

(i) $\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$, (ii) $\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, (iii) $\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}$

is in $\text{Span}\{\vec{u}, \vec{v}\}$ where

$$\vec{u} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} ?$$

- | | |
|--|--|
| <input type="checkbox"/> only (ii) and (iii) | <input type="checkbox"/> none of them. |
| <input type="checkbox"/> only (i) | <input type="checkbox"/> all of them |
| <input type="checkbox"/> only (i) and (ii) | <input type="checkbox"/> only (iii) |
| <input type="checkbox"/> only (ii) | <input checked="" type="checkbox"/> only (i) and (iii) |

8 Which of the following linear transformations is not onto? Only one answer applies.

- $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix}$
- $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 2x_1 + 4x_2 \end{bmatrix}$
- $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$
- $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - 3x_2 \\ -4x_1 + 6x_2 \end{bmatrix}$
- $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3x_1 - 2x_2 \\ x_1 - x_2 \end{bmatrix}$

Handwritten work for question 8:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix}$$



9 Let

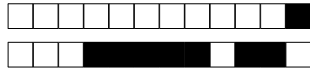
$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix},$$

and consider the system $A\mathbf{x} = \mathbf{0}$. How many free variables does the system have?

- 1
- 2
- 0
- 4
- 3

10 ♣ Which of the following statements are true? Choose all that apply.

- A set of two vectors is linearly independent if the vectors are not multiples of each other.
- Linear independence does not apply if a set contains only one vector.
- A set of 3 vectors in \mathbb{R}^5 must be linearly independent as long as they do not contain the zero vector.
- Any set containing the zero vector is linearly dependent.
- A set of 5 vectors in \mathbb{R}^3 can be linearly independent if none of the vectors is the zero vector, and none of the vectors are multiples of each other.



Part II: Justify your answer and show all work for full credit.

11 0 1 2 3 4 5 6 7 8 9 10 Administrative Use Only

Solve

$$\begin{aligned}x_1 - 3x_3 &= 8 \\2x_1 + 2x_2 + 9x_3 &= 7 \\x_2 + 5x_3 &= -2\end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array} \right] \quad \begin{aligned}x_1 &= 3(-1) + 8 = 5 \\x_2 &= -2 - 5(-1) = 3 \\x_3 &= -1\end{aligned}$$

Typo Calculation:

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 1 & 0 & 7 \\ 0 & 6 & 0 & -2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 6 & -9 \\ 0 & 6 & 0 & -2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 1 & 6 & -9 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 6 & -\frac{16}{3} \end{array} \right] \quad \begin{aligned}x_1 &= 3\left(-\frac{8}{9}\right) + 8 = \frac{16}{3} \\x_2 &= -\frac{1}{3} \\x_3 &= -\frac{8}{9}\end{aligned}$$

12 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*Find the *reduced row echelon form* for

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & 2 & 1 & 2 \\ 3 & 2 & 1 & -1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & 4 \\ 3 & 2 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & 1 & 0 \\ 0 & -4 & -2 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/3 & 0 \\ 0 & -4 & -2 & -7 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & -10/3 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 1 & 21/10 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & -1/10 \\ 0 & 1 & 0 & 7/10 \\ 0 & 0 & 1 & 21/10 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & 7/10 \\ 0 & 0 & 1 & 21/10 \end{bmatrix}$$



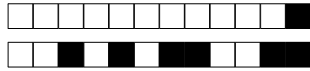
13 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Determine if the columns of the following matrix A span \mathbb{R}^4 .

$$A = \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 58 & 16 & 1 \\ 0 & 11 & -3 & 23 \end{bmatrix} \begin{array}{l} \\ 5R_1 + 7R_2 \\ -6R_1 + 7R_3 \\ R_1 + R_2 \end{array} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 58 & 16 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Only 3 pivots - not a span

14 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Let

$$A = \begin{bmatrix} 4 & 2 & 1 & -2 & 2 \\ 1 & 0 & -2 & 2 & 0 \\ -1 & 2 & -1 & -2 & -2 \end{bmatrix}.$$

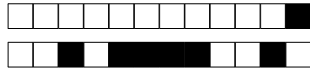
Find all solutions to the homogeneous system $Ax = 0$. Write your answer in parametric vector form.

$$\begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 4 & 2 & 1 & -2 & 2 \\ -1 & 2 & -1 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 0 & 2 & 9 & -10 & 2 \\ 0 & 2 & -3 & 0 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 9/2 & -5 & 1 \\ 0 & 0 & -12 & 10 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 9/2 & -5 & 1 \\ 0 & 0 & 1 & -5/6 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & -5/4 & 1 \\ 0 & 0 & 1 & -5/6 & 0 \end{bmatrix} \left(\begin{array}{l} -1/3 x_4 \\ 5/4 x_4 - x_5 \\ 5/6 x_4 \\ x_4 \\ x_5 \end{array} \right)$$

$$= x_4 \begin{pmatrix} -1/3 \\ 5/4 \\ 5/6 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$



15

 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be a linearly independent set. Prove that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ must be linearly independent.

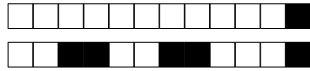
Suppose $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ is a LD set. Then, there are constants c_1, c_2, c_3 (at least 1 of which is not 0) for which

$$c_1 \underline{v}_1 + c_2 \underline{v}_2 + c_3 \underline{v}_3 = 0 \quad .$$

Therefore,

$$c_1 \underline{v}_1 + c_2 \underline{v}_2 + c_3 \underline{v}_3 + 0 \underline{v}_4 = 0,$$

and at least one of the coef. is not 0. Thus, $\{\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4\}$ is LD, a contradiction.

16 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 2x_1 - x_2 + x_3 \\ x_1 \\ x_1 + x_2 - 4x_3 \\ 3x_1 - 3x_2 + 2x_3 \end{bmatrix}.$$

I) Find the standard matrix of T .

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & -4 \\ 3 & -3 & 2 \end{bmatrix}$$

II) Is T one to one? Justify your answer.

Row reduction of the matrix gives

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -5 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix}$$

Each col is a pivot col., so T is 1-1

III) Is T onto? Justify your answer.

Not every row is a pivot row, so T is not onto.