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Math 313
Exam 1
May 12,13, 2015

Name: Key

Section:

Instructor:

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Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.
II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 4 points each.
III) Multiple choice questions that have more than one correct answer will be marked with a ♣. All other questions have only one correct answer.
IV) For questions which require a written answer, show all your work in the space provided and justify your answer.
V) Simplify your answers.
VI) Scientific calculators are allowed.
VII) No books or notes are allowed.
VIII) There is no time limit on this exam.

Part I: Multiple Choice Questions: *Questions marked with a ♣ have more than one correct answer. Mark **all** correct answers. The other questions have one right answer. (4 points each)*

1 For which value of the constant b does the system $x_1 + 3x_2 = 1$, $x_1 + bx_2 = -1$ have no solution?

- [ ] $b = -1$
- [ ] $b = 0$
- [ ] $b = 1$
- [ ] $b = -2$
- [x] $b = 3$
- [ ] $b = 2$

2 A row echelon form of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$ is

- [x] $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- [x] $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
- [ ] $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$

3 For an $m \times n$ matrix A and a vector $\mathbf{b}$ in $\mathbb{R}^m$, which of the following are logically equivalent to the statement: the columns of A span $\mathbb{R}^m$?

(i) Each $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of A.

(ii) Each row of A contains a pivot position.

(iii) The value of m is larger than the value of n.

- [ ] only (i)
- [x] only (i) and (ii)
- [ ] only (iii)
- [ ] only (ii)
- [ ] only (i) and (iii)
- [ ] none of them.
- [ ] all of them
- [ ] only (ii) and (iii)

4 Which of the following three statements about the matrix equation $A\vec{x} = \vec{b}$ are true?

(i) If A is a 4×2 matrix, then for every $\vec{b}$ in $\mathbb{R}^4$, the matrix equation $A\vec{x} = \vec{b}$ has a least one solution.

(ii) If A is a 2×4 matrix, then for every $\vec{b}$ in $\mathbb{R}^2$, the matrix equation $A\vec{x} = \vec{b}$ can never have a unique solution.

(iii) If A is a 3×3, then for every $\vec{b}$ in $\mathbb{R}^3$, the matrix equation $A\vec{x} = \vec{b}$ never has a solution.

- [ ] only (i) and (ii)
- [x] only (ii)
- [ ] none of them.
- [ ] only (i) and (iii)
- [ ] only (i)
- [ ] all of them
- [ ] only (iii)
- [ ] only (ii) and (iii)

5 ♣ Which of the following statements are true? (Assume any multiplication and additions are defined). Mark all that apply.

- [ ] If every row of A is a pivot row of A, then $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
- [x] A homogeneous system $A\mathbf{x} = \mathbf{0}$ is always consistent.
- [x] The system $A\mathbf{x} = \mathbf{b}$ can be inconsistent even though $A\mathbf{x} = \mathbf{0}$ is consistent.
- [ ] The system $A\mathbf{x} = \mathbf{b}$ can have a unique solution even though $A\mathbf{x} = \mathbf{0}$ does not.
- [ ] The system $A\mathbf{x} = \mathbf{0}$ can have a unique solution even though $A\mathbf{x} = \mathbf{b}$ does not.

6 ♣ Which of the following sets are linearly independent? Mark all that apply.

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 2 & -6 \end{bmatrix}$$

- [ ] $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} \right\}$
- [ ] $\left\{ \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$
- [ ] $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix} \right\}$
- [x] $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$
- [x] $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$

7 Which of the three vectors

$$\text{(i)} \begin{bmatrix}6\\2\\4\end{bmatrix}, \quad \text{(ii)} \begin{bmatrix}2\\-3\\1\end{bmatrix}, \quad \text{(iii)} \begin{bmatrix}4\\5\\0\end{bmatrix}$$

is in Span$\{\vec{u}, \vec{v}\}$ where

$$\vec{u} = \begin{bmatrix}3\\1\\2\end{bmatrix}, \quad \vec{v} = \begin{bmatrix}1\\4\\-2\end{bmatrix}?$$

- [ ] only (ii) and (iii)
- [ ] only (i)
- [ ] only (i) and (ii)
- [ ] only (ii)
- [ ] none of them.
- [ ] all of them
- [ ] only (iii)
- [x] only (i) and (iii)

8 Which of the following linear transformations is not onto? Only one answer applies.

- [ ] $T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}x_1 + x_2\\x_1 - x_2\end{bmatrix}$ $\quad \begin{bmatrix}1&1\\1&-1\end{bmatrix} \sim \begin{bmatrix}1&1\\0&-2\end{bmatrix}$
- [ ] $T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}x_1 + x_2\\2x_1 + 4x_2\end{bmatrix}$ $\quad \begin{bmatrix}1&1\\2&4\end{bmatrix} \sim \begin{bmatrix}1&1\\0&2\end{bmatrix}$
- [ ] $T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}x_2\\x_1\end{bmatrix}$ $\quad \begin{bmatrix}0&1\\1&0\end{bmatrix} \sim \begin{bmatrix}1&0\\0&1\end{bmatrix}$
- [x] $T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}2x_1 - 3x_2\\-4x_1 + 6x_2\end{bmatrix}$ $\quad \begin{bmatrix}2&-3\\-4&6\end{bmatrix} \sim \begin{bmatrix}2&-3\\0&0\end{bmatrix}$
- [ ] $T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}-3x_1 - 2x_1\\x_1 - x_2\end{bmatrix}$

9 Let

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix},$$

and consider the system $A\mathbf{x} = \mathbf{0}$. How many free variables does the system have?

- [x] 1
- [ ] 2
- [ ] 0
- [ ] 4
- [ ] 3

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

10 ♣ Which of the following statements are true? Choose all that apply.

- [x] A set of two vectors is linearly independent if the vectors are not multiples of each other.
- [ ] Linear independence does not apply if a set contains only one vector.
- [ ] A set of 3 vectors in $\mathbb{R}^5$ must be linearly independent as long as they do not contain the zero vector.
- [x] Any set containing the zero vector is linearly dependent.
- [ ] A set of 5 vectors in $\mathbb{R}^3$ can be linearly independent if none of the vectors is the zero vector, and none of the vectors are multiples of each other.

Part II: *Justify your answer and show all work for full credit.*

11 | ☐ **0** ☐ **1** ☐ **2** ☐ **3** ☐ **4** ☐ **5** ☐ **6** ☐ **7** ☐ **8** ☐ **9** ☐ **10** *Administrative Use Only*

Solve

$$\begin{array}{rcrcrcr} x_1 & & & - & 3x_3 & = & 8 \\ 2x_1 & + & 2x_2 & + & 9x_2 & = & 7 \\ & & x_2 & + & 5x_2 & = & -2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{bmatrix}$$

$x_1 = 3(-1)+8 = 5$

$x_2 = -2-5(-1) = 3$

$x_3 = -1$

Typo Calculation:

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 11 & 0 & 7 \\ 0 & 6 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 11 & 6 & -9 \\ 0 & 6 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & -1/3 \\ 0 & 11 & 6 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & -1/3 \\ 0 & 0 & 6 & -16/3 \end{bmatrix}$$

$x_1 = 3\left(\frac{-8}{9}\right)+8 = \frac{16}{3}$

$x_2 = -\frac{1}{3}$

$x_3 = -\frac{8}{9}$

12 □0 □1 □2 □3 □4 □5 □6 □7 □8 □9 □10 *Administrative Use Only*

Find the *reduced row echelon form* for

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & 2 & 1 & 2 \\ 3 & 2 & 1 & -1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & 4 \\ 3 & 2 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & 1 & 0 \\ 0 & -4 & -2 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/3 & 0 \\ 0 & -4 & -2 & -7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & -10/3 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 21/10 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & -1/10 \\ 0 & 1 & 0 & 7/10 \\ 0 & 0 & 1 & 21/10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & 7/10 \\ 0 & 0 & 1 & 21/10 \end{bmatrix}$$

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

Determine if the columns of the following matrix A span $\mathbb{R}^4$.

$$A = \begin{bmatrix} 7 & 2 & -5 & 8 \\ -5 & -3 & 4 & -9 \\ 6 & 10 & -2 & 7 \\ -7 & 9 & 2 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 58 & 16 & 1 \\ 0 & 11 & -3 & 23 \end{bmatrix} \begin{matrix} \\ 5R_1 + 7R_2 \\ -6R_1 + 7R_3 \\ R_1 + R_2 \end{matrix} \sim \begin{bmatrix} 7 & 2 & -5 & 8 \\ 0 & -11 & 3 & -23 \\ 0 & 58 & 16 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Only 3 pivots - not a span

14 □0 □1 □2 □3 □4 □5 □6 □7 □8 □9 □10 *Administrative Use Only*

Let

$$A = \begin{bmatrix} 4 & 2 & 1 & -2 & 2 \\ 1 & 0 & -2 & 2 & 0 \\ -1 & 2 & -1 & -2 & -2 \end{bmatrix}.$$

Find all solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$. Write your answer in parametric vector form.

$$\begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 4 & 2 & 1 & -2 & 2 \\ -1 & 2 & -1 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 0 & 2 & 9 & -10 & 2 \\ 0 & 2 & -3 & 0 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 9/2 & -5 & 1 \\ 0 & 0 & -12 & 10 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 9/2 & -5 & 1 \\ 0 & 0 & 1 & -5/6 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & -5/4 & 1 \\ 0 & 0 & 1 & -\frac{5}{6} & 0 \end{bmatrix} \qquad \begin{pmatrix} -1/3\, x_4 \\ 5/4\, x_4 - x_5 \\ 5/6\, x_4 \\ x_4 \\ x_5 \end{pmatrix}$$

$$= x_4 \begin{pmatrix} -1/3 \\ 5/4 \\ 5/6 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

15 □0 □1 □2 □3 □4 □5 □6 □7 □8 □9 □10 *Administrative Use Only*

Let $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ be a linearly independent set. Prove that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ must be linearly independent.

Suppose $\{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{v}_3\}$ is a LD set. Then, there are constants c_1, c_2, c_3 (at least 1 of which is not 0) for which

$$c_1 \underset{\sim}{v}_1 + c_2 \underset{\sim}{v}_2 + c_3 \underset{\sim}{v}_3 = 0 \quad .$$

Therefore,

$$c_1 \underset{\sim}{v}_1 + c_2 \underset{\sim}{v}_2 + c_3 \underset{\sim}{v}_3 + 0 v_4 = 0,$$

and at least one of the coef. is <u>not</u> 0. Thus, $\{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{v}_3, \underset{\sim}{v}_4\}$ is LD, a contradiction.

16 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation given by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 2x_1 - x_2 + x_3 \\ x_1 \\ x_1 + x_2 - 4x_3 \\ 3x_1 - 3x_2 + 2x_3 \end{bmatrix}.$$

I) Find the standard matrix of T.

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & -4 \\ 3 & -3 & 2 \end{bmatrix}$$

II) Is T one to one? Justify your answer.

Row reduction of the matrix gives

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -5 \\ 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix}$$

Each col is a pivot col., so T is 1-1

III) Is T onto? Justify your answer.

Not every row is a pivot row, so T is <u>not</u> onto.