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001

Math 313
Exam 2
May 26,27, 2015

Name: _____

Section: _____

Instructor: _____

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Instructions

- I) Do not write on the barcode area at the top of each page, or near the four circles on each page.
- II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 4 points each.
- III) Multiple choice questions that have more than one correct answer will be marked with a ♣. All other questions have only one correct answer.
- IV) For questions which require a written answer, show all your work in the space provided and justify your answer.
- V) Simplify your answers.
- VI) Scientific calculators are allowed.
- VII) No books or notes are allowed.
- VIII) There is no time limit on this exam.



Part I: Multiple Choice Questions: Questions marked with a ♣ have more than one correct answer. Mark **all** correct answers. The other questions have one right answer. (4 points each)

1 ♣ Let A , B and C be 3×5 , 5×3 and 5×5 matrices, respectively. Select the statements below that make sense. Mark all that apply.

- ABC
- $AC - (CB)^T$
- $AB - BA = 0$
- ACB^T
- BAC

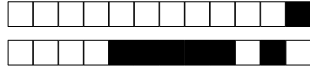
2 Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

What is the first entry of $A^{-1}\mathbf{v}$?

- 12
- 12
- 10
- 10
- 19
- 18

$$\begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 4 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & | & 1 & 0 & -3 \\ 0 & 1 & 0 & | & 0 & 1 & -4 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & | & 1 & -2 & 5 \\ 0 & 1 & 0 & | & 0 & 1 & -4 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -18 \\ \vdots \\ \vdots \end{bmatrix}$$



3 ♣ Let A and B be $n \times n$ matrices. Which of the following statements are always true? Mark all that apply.

- $A^T \mathbf{x} = \mathbf{b}$ can have more than one solution while $A\mathbf{x} = \mathbf{b}$ has only one.
- If A and B are invertible, AB is invertible, and $(AB)^{-1} = A^{-1}B^{-1}$.
- If $A\mathbf{x} = \mathbf{0}$ has only the solution $\mathbf{x} = \mathbf{0}$, then A is invertible.
- AB can be invertible when A is not.
- If A is invertible, the rows of A form a linearly independent set.

4 Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 5 \\ 7 & 8 & -3 \end{bmatrix}$$

find the cofactor C_{23} of A .

- 6
- 0
- 6
- 5
- 1
- 5

$$-(8 - 14) = 6$$



5 Let

$$A = \begin{bmatrix} 2 & 1 & 2 & 1 & 3 \\ 0 & -1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}.$$

Find $\det(A)$.

- 4
- 12
- 4
- 0
- 6
- 12
- 6

$$|A| = - \begin{vmatrix} 2 & 1 & 2 & 1 & 3 \\ 0 & -1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{vmatrix} = -(2(-1) \cdot 1 \cdot 2 \cdot 3) = 12$$

6 Let $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation whose standard matrix is A , and $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ a linear transformation whose standard matrix is B . If V is a parallelogram in \mathbb{R}^2 whose area is L , then the area of $T(S(V))$ is

- $L |\det(BA)|$
- L
- $L |\det B|$
- $|\det A \det B|$
- $L \det B \det A$
- $L \det A$



7 ♣ Which of the following subsets of \mathbb{R}^4 are subspaces of \mathbb{R}^4 ?

- $\{t\mathbf{v} : t \in \mathbb{R}\}$ where $\mathbf{v} \neq \mathbf{0}$
- $\{\mathbf{0}\}$
- $\{\mathbf{u}, \mathbf{v}\}$ where \mathbf{u} and \mathbf{v} are distinct nonzero vectors
- $\{t\mathbf{v} + s\mathbf{u} : t, s \in \mathbb{R}\}$ for $\mathbf{v} \neq \mathbf{0}$ and $\mathbf{u} \neq \mathbf{0}$
- $\{\mathbf{p} + t\mathbf{v} : t \in \mathbb{R}\}$ for $\mathbf{p} \neq \mathbf{0}$ and $\mathbf{v} \neq \mathbf{0}$
- $\{\mathbf{v} \in \mathbb{R}^4 : v_1 \geq 0\}$ where v_1 is the first entry of \mathbf{v}

8 The null space of a 4×5 matrix is a subspace of \mathbb{R}^k for

- $k = 1$
- $k = 3$
- $k = 4$
- $k = 5$
- $k = 2$
- $k = 6$



9 The reduced row echelon form of $A = \begin{bmatrix} 1 & 2 & 6 & 1 & -4 & 2 \\ 1 & 1 & 3 & 1 & -2 & 2 \\ 1 & 1 & 3 & 1 & -2 & 1 \end{bmatrix}$ is $U =$

$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Which columns of A form a basis for the column space of A ?

- Columns 1, 4, 6
- Columns 3, 5, 6
- Columns 1, 2, 6
- Columns 1, 2, 3
- Columns 2, 3, 5
- Columns 1, 2, 4

10 A basis for \mathbb{R}^2 is $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ where $\mathbf{b}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. The \mathcal{B} -coordinates of $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ are

$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

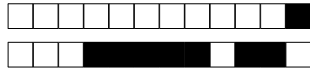
$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$
$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 5 & 5 \end{bmatrix} \begin{matrix} c_1 = 1 \\ c_2 = 1 \end{matrix}$$



Part II: *Justify your answer and show all work for full credit.*

11 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Let

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

a) Find C^{-1} or show that it does not exist.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix} \quad \text{Check:}$$
$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Solve the system $C\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

$$\underline{\mathbf{x}} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix}$$



12

 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Without calculating the inverses directly, use a property of the invertible matrix theorem to show whether the following matrices are invertible. Use a different property for each matrix, and state the property you are using.

a) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -3 - 1 = -4 \neq 0$$

Invertible because det is not 0.

b) $B = \begin{bmatrix} 3 & -7 & 2 \\ 1 & -1 & 5 \\ 1 & -5 & -8 \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 5 \\ 3 & -7 & 2 \\ 1 & -5 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 5 \\ 0 & -4 & -13 \\ 0 & -4 & -13 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 5 \\ 0 & -4 & -13 \\ 0 & 0 & 0 \end{bmatrix}$$

Only 2 pivots so not invertible



13

 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Find the determinant of

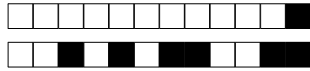
$$K = \begin{bmatrix} 0 & 0 & 0 & -1 & 3 \\ 0 & -2 & 0 & 1 & -1 \\ 3 & -2 & 0 & 0 & 1 \\ 0 & 2 & 3 & 1 & 0 \\ 0 & 0 & 1 & 2 & -4 \end{bmatrix}$$

$$3 \begin{vmatrix} 0 & 0 & -1 & 3 \\ -2 & 0 & 1 & -1 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & -4 \end{vmatrix} = \left(2 \begin{vmatrix} 0 & -1 & 3 \\ 3 & 1 & 0 \\ 1 & 2 & -4 \end{vmatrix} + 2 \begin{vmatrix} 0 & -1 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & -4 \end{vmatrix} \right) 3$$

$$= [2(-3 \begin{vmatrix} -1 & 3 \\ 2 & -4 \end{vmatrix} + 1 \begin{vmatrix} -1 & 3 \\ 1 & 0 \end{vmatrix}) + 2(\begin{vmatrix} -1 & 3 \\ 1 & -1 \end{vmatrix})]$$

$$= [2(-3(-2) - 3) + 2(-2)] 3$$

$$= (6 - 4) 3 = 6$$

14 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

The two parts of this question are unrelated.

a) Prove that if an $n \times n$ matrix A is invertible, then $\det A^{-1} = \frac{1}{\det A}$.

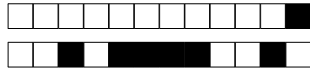
Since $A^{-1}A = I$,

$$1 = \det(I) = \det(A^{-1}A) = \det(A^{-1})\det(A)$$

Thus because $\det(A) \neq 0$, $\det(A^{-1}) = \frac{1}{\det(A)}$

b) For an $n \times n$ matrix A , prove that if $A^3 = 0$, then $\det A = 0$.

$$\det(A^3) = \det(A)\det(A)\det(A) = \det(A)^3 = 0, \text{ so } \det(A) = 0 \text{ (by taking the cube root of both sides.)}$$



15

 0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Find the adjugate (or classical adjoint) of the invertible matrix $A = \begin{bmatrix} 3 & 5 & 4 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$. Can you verify your answer? If so, how?

$$C_{11} = -1 \quad C_{12} = 1 \quad C_{13} = 1$$

$$|A| = C_{21} + C_{23} = -1 + 7 = 6$$

$$C_{21} = -1 \quad C_{22} = -5 \quad C_{23} = 7$$

$$\text{adj}(A) = \begin{bmatrix} -1 & -1 & 5 \\ 1 & -5 & 1 \\ 1 & 7 & -5 \end{bmatrix}$$

$$C_{31} = 5 \quad C_{32} = 1 \quad C_{33} = -5$$

$$\begin{bmatrix} -1 & -1 & 5 \\ 1 & -5 & 1 \\ 1 & 7 & -5 \end{bmatrix} \begin{bmatrix} 3 & 5 & 4 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$



16

0 1 2 3 4 5 6 7 8 9 10 *Administrative Use Only*

Find a basis for the null space of $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= -2x_2 - x_4 \\ x_3 &= -3x_4 \end{aligned}$$

$$\text{Basis} = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix} \right\}$$