NS 30318219 001

Math 313
Exam 2
May 26,27, 2015

Name:____________________

Section:____________________

Instructor:____________________

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Instructions

I) Do not write on the barcode area at the top of each page, or near the four circles on each page.
II) Fill in the correct boxes for your BYU ID and for the correct answer on the multiple choice completely. Multiple choice questions are 4 points each.
III) Multiple choice questions that have more than one correct answer will be marked with a ♣. All other questions have only one correct answer.
IV) For questions which require a written answer, show all your work in the space provided and justify your answer.
V) Simplify your answers.
VI) Scientific calculators are allowed.
VII) No books or notes are allowed.
VIII) There is no time limit on this exam.

Part I: Multiple Choice Questions: *Questions marked with a ♣ have more than one correct answer. Mark **all** correct answers. The other questions have one right answer. (4 points each)*

1 ♣ Let A, B and C be 3×5, 5×3 and 5×5 matrices, respectively. Select the statements below that make sense. Mark all that apply.

- [ ] ABC
- [x] $AC - (CB)^T$
- [ ] $AB - BA = 0$
- [ ] ACB^T
- [x] BAC

2 Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

What is the first entry of $A^{-1}\mathbf{v}$?

- [ ] -12
- [ ] 12
- [ ] 10
- [ ] -10
- [ ] -19
- [x] -18

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & 5 \\ 0 & 1 & 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -18 \\ \\ \\ \end{bmatrix}$$

3 ♣ Let A and B be $n \times n$ matrices. Which of the following statements are always true? Mark all that apply.

- [ ] $A^T\mathbf{x} = \mathbf{b}$ can have more than one solution while $A\mathbf{x} = \mathbf{b}$ has only one.
- [ ] If A and B are invertible, AB is invertible, and $(AB)^{-1} = A^{-1}B^{-1}$.
- [x] If $A\mathbf{x} = \mathbf{0}$ has only the solution $\mathbf{x} = \mathbf{0}$, then A is invertible.
- [ ] AB can be invertible when A is not.
- [x] If A is invertible, the rows of A form a linearly independent set.

4 Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 5 \\ 7 & 8 & -3 \end{bmatrix}.$$

find the cofactor C_{23} of A.

- [x] 6
- [ ] 0
- [ ] -6
- [ ] -5
- [ ] -1
- [ ] 5

$-(8-14)=6$

5 Let

$$A = \begin{bmatrix} 2 & 1 & 2 & 1 & 3 \\ 0 & -1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}.$$

Find $\det(A)$.

$$|A| = - \begin{vmatrix} 2 & 1 & 2 & 1 & 3 \\ 0 & -1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{vmatrix} = -(2(-1)\cdot 1\cdot 2\cdot 3) = 12$$

- [ ] -4
- [ ] -12
- [ ] 4
- [ ] 0
- [ ] -6
- [x] 12
- [ ] 6

6 Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation whose standard matrix is A, and $T : \mathbb{R}^2 \to \mathbb{R}^2$ a linear transformation whose standard matrix is B. If V is a parallelogram in $\mathbb{R}^2$ whose area if L, then the area of $T(S(V))$ is

- [x] $L\,|\det(BA)|$
- [ ] L
- [ ] $L\,|\det B|$
- [ ] $|\det A \det B|$
- [ ] $L \det B \det A$
- [ ] $L \det A$

7 ♣ Which of the following subsets of $\mathbb{R}^4$ are subspaces of $\mathbb{R}^4$?

- [x] $\{t\mathbf{v} : \mathbf{t} \in \mathbb{R}\}$ where $\mathbf{v} \neq \mathbf{0}$
- [x] $\{\mathbf{0}\}$
- [ ] $\{\mathbf{u}, \mathbf{v}\}$ where $\mathbf{u}$ and $\mathbf{v}$ are distinct nonzero vectors
- [x] $\{t\mathbf{v} + s\mathbf{u} : t, s \in \mathbb{R}\}$ for $\mathbf{v} \neq \mathbf{0}$ and $\mathbf{u} \neq \mathbf{0}$
- [ ] $\{\mathbf{p} + t\mathbf{v} : t \in \mathbb{R}\}$ for $\mathbf{p} \neq \mathbf{0}$ and $\mathbf{v} \neq \mathbf{0}$
- [ ] $\{\mathbf{v} \in \mathbb{R}^4 : v_1 \geq 0\}$ where v_1 is the first entry of $\mathbf{v}$

8 The null space of a 4×5 matrix is a subspace of $\mathbb{R}^k$ for

- [ ] $k = 1$
- [ ] $k = 3$
- [ ] $k = 4$
- [x] $k = 5$
- [ ] $k = 2$
- [ ] $k = 6$

9 The reduced row echelon form of $A = \begin{bmatrix} 1 & 2 & 6 & 1 & -4 & 2 \\ 1 & 1 & 3 & 1 & -2 & 2 \\ 1 & 1 & 3 & 1 & -2 & 1 \end{bmatrix}$ is $U = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Which columns of A form a basis for the column space of A?

- [ ] Columns 1, 4, 6
- [ ] Columns 3, 5, 6
- [x] Columns 1, 2, 6
- [ ] Columns 1, 2, 3
- [ ] Columns 2, 3, 5
- [ ] Columns 1, 2, 4

10 A basis for $\mathbb{R}^2$ is $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ where $\mathbf{b}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. The $\mathcal{B}$-coordinates of $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ are

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 5 & 5 \end{bmatrix} \quad \begin{matrix} c_1 = 1 \\ c_2 = 1 \end{matrix}$$

- [ ] $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- [ ] $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$
- [ ] $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- [ ] $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$
- [ ] $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- [x] $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Part II: *Justify your answer and show all work for full credit.*

11 ☐ **0** ☐ **1** ☐ **2** ☐ **3** ☐ **4** ☐ **5** ☐ **6** ☐ **7** ☐ **8** ☐ **9** ☐ **10** *Administrative Use Only*

Let

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

a) Find C^{-1} or show that it does not exist.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}$$

Check:

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Solve the system $C\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$.

$$\underline{x} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix}$$

12 □0 □1 □2 □3 □4 □5 □6 □7 □8 □9 □10 *Administrative Use Only*

Without calculating the inverses directly, use a property of the invertible matrix theorem to show whether the following matrices are invertible. Use a different property for each matrix, and state the property you are using.

a) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -3 - 1 = -4 \neq 0$$

Invertible because det is not 0.

b) $B = \begin{bmatrix} 3 & -7 & 2 \\ 1 & -1 & 5 \\ 1 & -5 & -8 \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 5 \\ 3 & -7 & 2 \\ 1 & -5 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 5 \\ 0 & -4 & -13 \\ 0 & -4 & -13 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 5 \\ 0 & -4 & -13 \\ 0 & 0 & 0 \end{bmatrix}$$

Only 2 pivots so not invertible

13 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

Find the determinant of

$$K = \begin{bmatrix} 0 & 0 & 0 & -1 & 3 \\ 0 & -2 & 0 & 1 & -1 \\ 3 & -2 & 0 & 0 & 1 \\ 0 & 2 & 3 & 1 & 0 \\ 0 & 0 & 1 & 2 & -4 \end{bmatrix}.$$

$$3\begin{vmatrix} 0 & 0 & -1 & 3 \\ -2 & 0 & 1 & -1 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & -4 \end{vmatrix} = \left(2\begin{vmatrix} 0 & -1 & 3 \\ 3 & 1 & 0 \\ 1 & 2 & -4 \end{vmatrix} + 2\begin{vmatrix} 0 & -1 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & -4 \end{vmatrix}\right)3$$

$$= \left[2\left(-3\begin{vmatrix} -1 & 3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} -1 & 3 \\ 1 & 0 \end{vmatrix}\right) + 2\left(\begin{vmatrix} -1 & 3 \\ 1 & -1 \end{vmatrix}\right)\right]$$

$$= [2(-3(-2)-3)+2(-2)]3$$

$$= (6-4)3 = 6$$

14 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

The two parts of this question are unrelated.

a) Prove that if an $n \times n$ matrix A is invertible, then $\det A^{-1} = \dfrac{1}{\det A}$.

Since $A^{-1}A = I$,

$1 = \det(I) = \det(A^{-1}A) = \det(A^{-1})\det(A)$

Thus because $\det(A) \neq 0$, $\det(A^{-1}) = \dfrac{1}{\det(A)}$

b) For an $n \times n$ matrix A, prove that if $A^3 = 0$, then $\det A = 0$.

$\det(A^3) = \det(A)\det(A)\det(A) = \det(A)^3 = 0$, so $\det(A) = 0$ (by taking the cube root of both sides.)

15 ☐0 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

Find the adjugate (or classical adjoint) of the invertible matrix $A = \begin{bmatrix} 3 & 5 & 4 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$. Can you verify your answer? If so, how?

$C_{11} = -1 \quad C_{12} = 1 \quad C_{13} = 1$

$C_{21} = -1 \quad C_{22} = -5 \quad C_{23} = 7$

$C_{31} = 5 \quad C_{32} = 1 \quad C_{33} = -5$

$|A| = C_{21} + C_{23} = -1 + 7 = 6$

$$\operatorname{adj}(A) = \begin{bmatrix} -1 & -1 & 5 \\ 1 & -5 & 1 \\ 1 & 7 & -5 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 5 \\ 1 & -5 & 1 \\ 1 & 7 & -5 \end{bmatrix} \begin{bmatrix} 3 & 5 & 4 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

16 □0 □1 □2 □3 □4 □5 □6 □7 □8 □9 □10 *Administrative Use Only*

Find a basis for the null space of $A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -2x_2 - x_4$
$x_3 = -3x_4$

$$\text{Basis} = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -3 \\ 1 \end{pmatrix} \right\}$$