Name:____

Student ID:_____________________________________

Instructor: Steven McKay

Math 313-11 (Linear Algebra) Practice Exam 1

Instructions:

- For questions which require a written answer, show all your work. Full credit will be given only if the necessary work is shown justifying your answer.
- Simplify your answers.
- Scientific calculators are allowed.
- Should you have need for more space than is allocated to answer a question, use the back of the page the problem is on and indicate this fact.
- Please do not talk about the test with other students until after the last day to take the exam.

Part I: Multiple Choice Questions: *Mark all answers which are correct for each question.*

1. Consider the following augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 5 \\ 3 & 3 & 2 & 1 \\ 4 & 0 & -2 & b \end{array}\right]$$

   For which value of b is the system consistent?

   a) -5

   b) 3

   c) -8

   d) 4

   e) 5

   f) It is consistent for all values of b.

   g) It is inconsistent for all values of b.

2. Consider the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{array}\right]$$

   Which of the following statements are true? (Mark all that apply)

   a) The system is overdetermined.

   b) The system is underdetermined.

   c) The system is inconsistent.

   d) There is an equivalent row echelon form having a pivot in every column of the system matrix.

   e) The system has infinitely many solutions.

   f) There are no free variables.

   g) The system has a unique solution.

3. Which matrix product is defined?

a) $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

4. Let

$$\mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \qquad \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Which of the following sets are **linearly dependent**? Select all that apply.

a) $\{\mathbf{0}\}$

b) $\{\mathbf{v}\}$

c) $\{\mathbf{w}, \mathbf{0}\}$

d) $\{\mathbf{v}, \mathbf{0}\}$

e) $\{\mathbf{v}, \mathbf{w}\}$

f) $\{\mathbf{v}, \mathbf{w}, \mathbf{0}\}$

5. Define $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$ by the following matrix

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ -1 & -3 \end{bmatrix}.$$

Which of the following vectors is not in the range of T?

a) $\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$

b) $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$

d) $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

e) $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

6. For a linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^3$ it is known that it maps the vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ respectively to the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Which of the following statements are true?

a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

b) There can be only one transformation T satisfying the above condition.

c) T is one-to-one.

d) T is onto.

Part II: *Fill in the blank with the* ***best*** *possible answer. (4 points each)*

7. If a system of linear equations has a solution, we say it is ___________________________.

8. The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution

if and only if ____.

9. Every linear transformation can be represented by a ___________________________.

Part III: *Free response: Justify your answer and show all work for full credit.*

10. Find all solutions to the system of equations

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 1 \\ 2x_1 + x_3 + x_4 &= 0 \\ x_1 + 2x_2 + x_3 + 2x_4 &= 1 \\ x_1 + x_2 &= -1 \end{aligned}$$

11. Describe the steps of the *backwards* phase of the *elimination algorithm*. Under which conditions on an augmented matrix is it used? What is the purpose of the algorithm? When does the algorithm terminate?

12. Is $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ in Span $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \right\}$? Justify your answer.

13. Do the vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$ span $\mathbb{R}^3$? Prove or disprove.

14. Consider the matrix equation $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & -1 & 3 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}.$$

Find all solutions to this equation, and express them in parametric form.

15. Which conditions on the entries of a two by two standard matrix of a transformation must hold if T corresponds to a rotation in the plane?

16. Indicate whether the statement is always true or sometimes false. Provide a complete detailed proof for true statements or a counterexample for false statements.

a) If $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent, then $\{2\mathbf{x}, \mathbf{x} + \mathbf{y}\}$ is linearly independent.

b) If T and $S : \mathbb{R}^n \to \mathbb{R}^m$ are linear transformations, and let $\mathbf{v}, \mathbf{u}, \mathbf{w} \in \mathbb{R}^n$ with $T(\mathbf{v}) = S(\mathbf{v})$, $T(\mathbf{u}) = S(\mathbf{u})$, and $T(\mathbf{w}) = S(\mathbf{w})$. Then $T(\mathbf{x}) = S(\mathbf{x})$ for all $\mathbf{x}$ in $\text{Span}\{\mathbf{v}, \mathbf{u}, \mathbf{w}\}$.

END OF EXAM