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Name:____________________________

Section:___________________________

Instructor: Steven M. McKay

Math 314
Exam 3
June 12,13,14

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Instructions

- Do not write on the barcode area at the top of each page, or near the four circles on each page.
- Mark the correct choice in the multiple choice section. Each multiple question is worth 4 points.
- Show your work in the free response section. Correct answers without appropriate work will receive no credit.
- Work in the free response section must be given in the space provided, or on the back of the sheet the problem is on. If you use the back of the page, make sure you write ”work on back” or equivalent.
- Scientific calculators are allowed. Graphing calculators are not. No other electronic device is allowed.

Part 1: Multiple Choice Check the correct response.

1 Did you watch or read Flatland?

- ☐ No
- ☐ Yes

2 Find the area inside the Limacon $r = 2 + cos(\theta)$ $(0 \leq \theta \leq 2\pi)$.

- ☐ $\frac{3}{2}\pi$
- ☐ 2π
- ☐ $\frac{7}{2}\pi$
- ☐ 3π
- ☐ 0
- ☐ $\frac{9}{2}\pi$

3 Find the centroid of the quarter disk of radius 2 in the first quadrant. Choose the correct answer for $\overline{x}$.

- ☐ $\frac{4}{3\pi}$
- ☐ 1
- ☐ $\frac{1}{\pi}$
- ☐ $\frac{8}{3\pi}$
- ☐ $\frac{8}{3}$
- ☐ $\frac{4}{3}$

4 Find $\iiint_T x\,dV$ where T is the region bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

- ☐ $\frac{1}{2}$
- ☐ $\frac{1}{12}$
- ☐ $\frac{1}{24}$
- ☐ $\frac{1}{8}$
- ☐ 1
- ☐ $\frac{1}{6}$

5 A hole of radius 1 in. is drilled through the center of a solid sphere of radius 3 in. Find the volume of the material removed.

- ☐ $\frac{2}{3}\pi\ in^3$
- ☐ $\frac{104}{3}\pi\ in^3$
- ☐ $\frac{4}{3}\pi\ in^3$
- ☐ $\frac{22}{3}\pi\ in^3$
- ☐ $\frac{8}{3}\pi\ in^3$
- ☐ $\frac{57}{3}\pi\ in^3$

6 A ball consists of a hollow sphere with inner radius of 10 mm and outer radius of 12 mm. the density is given by

$$\delta(x, y, z) = 0.05\sqrt{x^2 + y^2 + z^2}$$

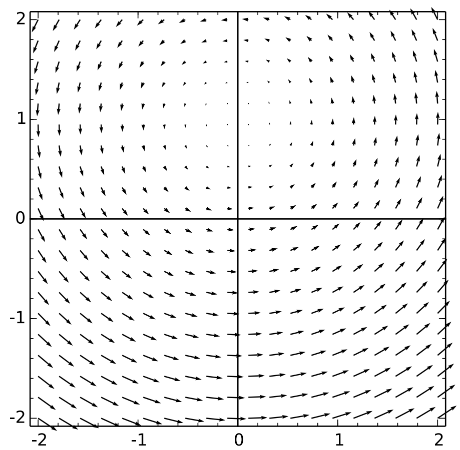
in grams per cubic millimeter. Find the mass of the ball in grams. (Choose the answer that is the closest.)

- ☐ 1670
- ☐ 1680
- ☐ 1692
- ☐ 1696
- ☐ 1676
- ☐ 1686

7 If $u = x^2 + y^2$ and $v = x^2 - y^2$, write x and y in terms of u and v, and find $\frac{\partial(x, y)}{\partial(u, v)}$ at $u = 1$ and $v = 0$. (Note: the sign could be positive or negative depending on choices for u and v.)

- ☐ $\pm\frac{1}{4}$
- ☐ ± 1
- ☐ $\pm\frac{1}{2}$
- ☐ $\pm\frac{1}{8}$
- ☐ $\pm\frac{1}{16}$
- ☐ 0

8 The following is the plot of which vector field?



- ☐ $\langle 1-y, x\rangle$
- ☐ $\langle -x, y\rangle$
- ☐ $\langle x, y\rangle$
- ☐ $\langle x, x\rangle$
- ☐ $\langle x^2, y\rangle$
- ☐ $\langle x+y, x-y\rangle$

9 Set up $\oint_C x\, ds$ where C is parametrized by $\mathbf{r(t)} = \langle \mathbf{t^2}, \mathbf{t^3}\rangle$, $0 \le t \le 1$.

- ☐ $\int_0^1 t^3 \cdot 2t\, dt$
- ☐ $\int_0^1 t^3\sqrt{4+9t^2}\, dt$
- ☐ $\int_0^1 t^2\sqrt{1+9t^4}\, dt$
- ☐ $\int_0^1 t\sqrt{1+t^6}\, dt$
- ☐ $\int_0^1 t^3\sqrt{1+t^6}\, dt$
- ☐ $\int_0^1 t^2 \cdot 3t^2\, dt$

10 Which of the following vector fields is conservative? (There is only one correct choice).

- ☐ $\langle x \ln y, \dfrac{x}{y} \rangle$
- ☐ $\langle x^2y - y^3, \dfrac{x^3}{3} - 3xy^2 \rangle$
- ☐ $\langle x, xy - 1 \rangle$
- ☐ $\langle ye^x, xe^y \rangle$
- ☐ $\langle y^2 - 2y, x^2 - 2x \rangle$
- ☐ $\langle x - y, x + y \rangle$

11 Use Green's Theorem to rewrite

$$\oint_C x^2y\,dx + xy^2\,dy$$

as a double integral over R (the interior of the simple closed curve C).

- ☐ $\displaystyle\iint_R x^2 + y^2\,dA$
- ☐ $\displaystyle\iint_R y^2 - x^2\,dA$
- ☐ $\displaystyle\iint_R x^2y - xy^2\,dA$
- ☐ $\displaystyle\iint_R xy^2 + x^2y\,dA$
- ☐ $\displaystyle\iint_R x^2 - y^2\,dA$
- ☐ $\displaystyle\iint_R xy^2 - x^2y\,dA$

Part II: Free Response. Show all work for each question in the space provided. If necessary, write "work on back" at the bottom of the space, and continue working on the back of the page. **DO NOT** write in the administrative areas.

12 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

Find the center of mass of an object whose position is the triangle with vertices $(0, 0)$, $(2, 0)$, and $(2, 1)$ if the density is given by $\delta(x, y) = y$.

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13 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

Find $\iint_R xy\, dA$ where R is the region in the first quadrant bounded between $xy = 1$, $xy = 2$, $\frac{y}{x} = 1/2$, and $\frac{y}{x} = 1$.

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14 ☐1 ☐2 ☐3 ☐4 ☐5 ☐6 ☐7 ☐8 ☐9 ☐10 *Administrative Use Only*

Find $\oint_C (\sqrt{x}+y^2)\,dx+(x^2-\sqrt{y})\,dy$ where C is the arc of the curve $y = sinx$ from $(0,0)$ to $(\pi,0)$ and the line segment from $(\pi,0)$ to $(0,0)$. (**Hint:** Use Green's Theorem.)

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15 $\square$ **1** $\square$ **2** $\square$ **3** $\square$ **4** $\square$ **5** $\square$ **6** $\square$ **7** $\square$ **8** $\square$ **9** $\square$ **10** *Administrative Use Only*

Let

$$\mathbf{F}(\mathbf{x}, \mathbf{y}) = \langle \mathbf{2x} + \mathbf{2xy}, \mathbf{x^2} + \sin(\mathbf{y}) \rangle.$$

a) Find the scalar potential f (that is the function with $\nabla f = \mathbf{F}$).

b) Find $\oint_C \mathbf{F} \cdot \mathbf{dr}$ where C is any cruve from $(1, 1)$ to $(2, 0)$.

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16 □1 □2 □3 □4 □5 □6 □7 □8 □9 □10 *Administrative Use Only*

Find $\iiint\limits_T e^{\sqrt{x^2+y^2+z^2}}\,dV$ where T is the volume inside the sphere $x^2+y^2+z^2=4$ and below the cone $z=\sqrt{x^2+y^2}$.

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