Math 521 Lecture #1
§1.1.1: The Program of Applied Mathematics

What is Applied Mathematics?

In one sense, it is the set of methods we use to solve equations that arise in science, engineering, and other areas.

Traditionally, these methods are techniques we use to solve differential equations.

In another sense, Applied Mathematics is applied analysis, i.e., the theory that underlies the methods we use.

We may say that Applied Mathematics is about those techniques useful in the mathematical analysis of real-word phenomena.

In a broader sense, Applied Mathematics is about **mathematical modeling**.

A **mathematical model** is an equation, or set of equations, that describes some physical problem or phenomenon having its origins in science, engineering, etc.

Example. Consider the (hopefully familiar) mathematical model of the unforced mass-spring system,

$$mu'' + \gamma u' + ku = 0,$$

where u is the displacement of an object from its rest position, m is the mass of the object, γ is the damping constant of the medium the object is moving in, and k is the spring constant.

The problem that led to this model is that of determining the position of the object at any time t.

Several physical laws were used to derive the ordinary differential equation for the unforced mass-spring system: Newton's second law that the sum of the forces acting on the object equals mu'', a linear drag law that gives $F_d = -\gamma u'$ for $\gamma \geq 0$, and Hooke's Law that gives $F_s = -ku$ for $k \geq 0$.

There is a method to solve the second-order linear ODE (i.e., guessing $u = c_1 e^{r_1 t} + c_2 e^{r_2 t}$) that gives explicit solutions depending on the initial conditions of position and velocity.

The nature of the solutions depends on the values of parameters m, γ, and k: when $\gamma > 0$, however, all solutions, regardless of the initial conditions, tend to 0 as $t \to \infty$.

Comparison with laboratory experiments of the mass-spring system with the predictions of the model then determine if the model sufficiently quantifies the motion, or if modifications to the model are needed.

It might be that the linear drag law is not a good fit with experimental data, and so we might try a quadratic drag law instead, leading to the ODE

$$mu'' + \gamma u'|u'| + ku = 0.$$

Now solving the ODE has become much harder, but we reasonably believe that solutions still depend on the values of the parameters m, γ, and k.

This example illustrates the various stages of the modeling process: there is a physical problem having its origins in science, engineering, etc., the formulation of the problem into equations, the solution of the equations (either analytic or numerical), comparison of the solutions with empirical data, and modification of the model.

Applied Mathematics deals with all the stages of mathematically modeling.

In this course, we will focus on the formulation and solutions of a mathematical model.

Our analysis begins next lecture with dimensional analysis, next scaling, and finally onto solutions both exact and approximate.