



Department of Mathematics First Annual Math Contest Afternoon Session: Problems

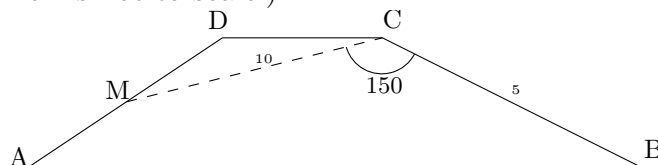
Problem 1B

Give an exact value for the integral

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx.$$

Problem 2B

Let $ABCD$ be a trapezoid, with sides AB and CD parallel. Let M be the midpoint of side AD . Assume that angle BCM is 150 degrees. Suppose that the length of BC is 5 and the length of CM is 10. Find the area of the trapezoid $ABCD$. (See the diagram below, which is not to scale.)



Problem 3B

Find the smallest n such that any sequence a_1, \dots, a_n whose values are pairwise relatively prime square-free integers between 2 and 2006 must contain a prime. (An integer is square-free if it is not divisible by any square except 1. Two integers are relatively prime if their greatest common divisor is 1.)

Problem 4B

For a given integer n , define $p(n)$ to be the product of the nonzero digits of n . For instance $p(5) = 5$, $p(2006) = 12$, and $p(1024) = 8$. Compute the exact value of the sum

$$p(1) + p(2) + p(3) + p(4) + \dots + p(999999)$$

Problem 5B

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function with infinitely many derivatives, such that $f(1) = 2$ and for all $\alpha, \beta \in \mathbb{R}$ with $\alpha^2 + \beta^2 = 1$, $f(\alpha)f(\beta) = f(x)$ for all x . Find $f(x)$.