



## Department of Mathematics First Annual Math Contest Morning Session: Problems

### Problem 1A

Compute the exact value of

$$\int_{2002}^{2006} \sqrt{(x - 2002)(2006 - x)} dx.$$

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### Problem 2A

200 students took an exam with 6 questions. Every question was correctly answered by at least 120 students. Show that there must be two students such that every question was correctly answered by at least one of them.

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### Problem 3A

Let  $r_1$  and  $r_2$  be two roots of  $f(x) = x^4 - x^3 + ax^2 - 8x - 8$ . Suppose that  $r_1 r_2 = 2$ . Find the value of  $a$ . (You may not assume that any of  $r_1, r_2$ , or  $a$  are integers, or even real numbers.)

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### Problem 4A

Give the exact value of

$$\cos\left(\frac{\pi}{5}\right) \cos\left(\frac{2\pi}{5}\right) \cos\left(\frac{3\pi}{5}\right) \cos\left(\frac{4\pi}{5}\right).$$

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### Problem 5A

Evaluate the infinite series

$$S = 1 - \frac{2^3}{1!} + \frac{3^3}{2!} - \frac{4^3}{3!} + \frac{5^3}{4!} - \dots$$