2009 Graduate Workshop on Zeta functions, L-functions and their applications

June 1-4, 2009
Utah Valley University
Orem, Utah

Reimbursement

For graduate students who were promised funding and need reimbursements for expenses, reimbursement forms will be available from the organizers. Please fill them out and return them as soon as possible.

Wireless Access

To access the internet use the wireless network “uvu guest”. You will be propted for a login and a password: for the login use “UVU” and for the password use “university”. If you have any problems, contact the help desk (801)863-8888 or speak to one of the conference organizers.

Restauraunts near campus

There is a cafeteria in the building very near the meeting room. For dinner, you may wish to go a bit further. Here are some off-campus options, in order of distance from the university.

- **IHOP** 850 W. University Parkway, right next to the Hampton Inn
- **McGrath's Fish House** 860 W. University Parkway, right next to the Hampton Inn
  A bit expensive, but the food is good.
- **Thai Evergreen** 1360 Sandhill Rd., across from Walmart
- **Applebee's Neighborhood Grill** 290 W. University Parkway
- **Golden Corral Buffet** 225 W. University Parkway
- **Fuddrucker's** 210 W. University Parkway
- **Chili's Grill and Bar** 122 E. University Parkway
- **Outback Steakhouse** 372 E. University Parkway
- **Chick-Fil-A** 575 E. University Parkway
- **The Old Spaghetti Factory** 575 E University Parkway
- **P.F. Chang's China Bistro** 575 E. University Parkway
- **Carraba's Italian Grill** 583 E. University Parkway
- **Goodwood Barbecue** 777 E. University Parkway

Note: the last five restaraunts are located in or near University Mall, about two miles from the University.

Monday June 1, 2009

9:00-9:30 AM	Welcome and Introductory Remarks
9:30-10:30 AM	**Brian Conrey** American Institute of Mathematics Random matrix theory and analytic number theory, I
10:30-11:00 AM	Break
11:00-12:00 AM	**Ram Murty** Queens University Introduction to Artin L-functions
12:00-2:00 PM	Lunch Break
2:00-3:00 PM	**Steve Gonek** University of Rochester An Introduction to the theory of the Riemann Zeta-Function, I
3:00-3:30 PM	Break
3:30-3:55 PM	**Ji Bian** University of Rochester Pair Correlation and Derivatives of Riemann Xi-Function
4:00-4:25 PM	**Andrew Ledoan** University of Rochester Zeros of partial sums of the Riemann zeta-function
4:30-4:55 PM	**Andreas Weingartner** Southern Utah University Zeros of Dirichlet series with periodic coefficients
5:00-5:30 PM	**Joseph Hundley** Southern Illinois University Integral Representations: What we've found, where we're looking, and why we're looking there

Tuesday, June 2, 2009

9:00-10:00 AM	**Brian Conrey** American Institute of Mathematics Random matrix theory and analytic number theory, II
10:05–10:30 AM	**Tim Gillespie** University of Iowa A prime number theorem for Rankin-Selberg L-functions over different fields
10:30-11:00 AM	Break
11:00-12:00 AM	**Ram Murty** Queens University The Chebotarev density theorem
12:00-2:00 PM	Lunch Break
2:00-3:00 PM	**Steve Gonek** University of Rochester An introduction to the theory of the Riemann Zeta-Function, II
3:00-3:30 PM	Break
3:30-3:55 PM	**Sidney Graham** Central Michigan University The Ideal Sieve
4:00-4:50 PM	**Steven J. Miller** Williams College Topics in L-functions and random matrix theory.
5:00-5:25	**Shenghao Sun** University of California at Berkeley Zeta functions of Artin stacks over finite fields
7:00-9:00	Conference Banquet

Wednesday, June 3, 2009

9:00-10:00 **Brian Conrey** American Institute of Mathematics
Random matrix theory and analytic number theory, III

10:15-11:15 **Ram Murty** Queens University
Special values of Artin *L*-series

11:30-12:30 **Steve Gonek** University of Rochester
An Introduction to the Theory of the Riemann Zeta-Function, III

FREE AFTERNOON

Thursday, June 4, 2009

9:00-10:00 **Brian Conrey** American Institute of Mathematics
Random matrix theory and analytic number theory, IV

10:05-10:30 **Dimitrios Koukoulopoulos**
University of Illinois at Urbana-Champaign
Generalized Multiplication Tables

10:30-11:00 Break

11:00-12:00 **Ram Murty** Queens University
L-series and transcendental numbers

12:00-2:00 Lunch Break

2:00-3:00 **Steve Gonek** University of Rochester
An Introduction to the Theory of the Riemann Zeta-Function, IV

3:00-3:30 Break

3:30-3:55 **Jeremy Rouse** University of Illinois at Urbana-Champaign
Bounds for the coefficients of powers of the Δ-function

4:00-4:25 **Takashi Nakamura** Tokyo University of Science
The joint universality and the generalized strong-recurrence for L-functions

4:30-4:55 **Ameya Pitale** University of Oklahoma
Special values of L-functions

5:00-5:25 **Mohammad Zaki** University of Illinois at Urbana-Champaign
Dirichlet series for weighted convolutions of von Mangoldt function

Abstracts

Ji Bian

Pair Correlation and Derivatives of Riemann Xi-Function

Abstract: In this talk, we describe applications of Montgomery's method in the study of the pair correlation function of the zeros of $\xi^{(\kappa)}(s)$, the κth derivative of Riemann's xi-function. In particular, we find a new method to estimate upper bounds of the percentage of multiple zeros of $\zeta(s)$. Our work also suggests that the zeros of $\xi^{(\kappa)}(s)$ tend to even out when we take high derivative, a fact predicted by a general theorem of Farmer and Rhoades. We explore this and also obtain new results on the size of small gaps between the zeros of $\xi^{(\kappa)}(s)$ and on the percentage of simple zeros of $\xi^{(\kappa)}(s)$.

Brian Conrey Random matrix theory and analytic number theory (4 talks)

Abstract: Recently there has been an increased understanding of how to use statistics involving groups of random matrices to model various local statistics (i.e. statistics of zeros and their spacings) and global statistics (moments and the value distributuion) of families of L-functions. In these talks we will develop the basic tools of Random Matrix Theory and show how to adapt these tools to compute very precise statistics of L-functions.

Tim Gillespie

A prime number theorem for Rankin-Selberg L-functions over number fields

Abstract: We compute a prime number theorem for a Rankin-Selberg L-function associated to two automorphic cuspidal representations π of $GL_n(\mathbb{A}_E)$ and σ of $GL_m(\mathbb{A}_F)$ where E and F are finite Galois extensions. When E is not necessarily equal to F we must reduce to the case when E and F are cyclic extensions with π and σ invariant under the Galois action (i.e. $\pi^\gamma \cong \pi$ for all γ in $Gal(E/\mathbb{Q})$), to exploit a result of Arthur and Clozel which gives that

$$L(s,\pi) = L(s,\pi_{\mathbb{Q}})L(s,\pi_{\mathbb{Q}} \otimes \eta_{E/\mathbb{Q}})...L(s,\pi_{\mathbb{Q}} \otimes \eta_{E/\mathbb{Q}}^{\ell-1})$$

where $\pi_{\mathbb{Q}}$ is an automorphic cuspidal representation of $GL_n(\mathbb{A}_{\mathbb{Q}})$, $\eta_{E/\mathbb{Q}}$ is an idele class character associated to E by class field theory, and ℓ denotes the degree of E over $\mathbb{Q}$. Such estimates are useful in computing the n-level correlation function of normalized nontrivial zeroes of a product of L-functions associated to cuspidal representations of GL_m.

Steve Gonek

An Introduction to the Theory of the Riemann Zeta-Function (Four Lectures)

Abstract: The purpose of these lectures is to provide an introduction to the theory of the Riemann zeta-function. The first lecture will be a survey talk in which we describe the contents of Riemann's pivotal 1859 paper "On the number of primes less than a given magnitude", the early work it spurred in the theory of the zeta-function, and how the theory developed subsequently down to the present day. In the second lecture we examine one of the most important tools of analytic number theory, mean value theorems. We show how they have been used to achieve a number of the important results mentioned in the first lecture. The final two talks will focus on two main areas in the subject and cover them in some depth.

Sidney Graham

The Ideal Sieve

Abstract: I will discuss a simplified sieving problem in which all the sifting primes p lie in the interval $z^\alpha < p \leq z^\beta$. For certain values of α and β, we can construct optimal upper and lower bound sieves.
This is joint work with Hugh Montgomery.

Joseph Hundley

Integral Representations: What we've found, where we're looking, and why we're looking there

Abstract: I shall attempt to survey some of the ideas and results in the theory of integral representations of "Rankin-Selberg" type, with an emphasis on my ongoing work with David Ginzburg searching for new integral representations.

Dimitrios Koukoupolous

Generalized multiplication tables

Abstract: For integers $k \geq 2$ and $N_1, \ldots, N_k$ consider the k-dimensional multiplication table formed by taking all products $n_1 \cdots n_k$ with $n_i \leq N_i$, $1 \leq i \leq k$. Let $A_k(N_1, \ldots, N_k)$ be the number of distinct integers that appear in this table. We seek bounds on $A_k(N_1, \ldots, N_k)$. In 2004 Ford established the order of magnitude of $A_2(N, N)$. We generalize Ford's result by determining the order of magnitude of $A_k(N, \ldots, N)$ when $k \geq 3$. Finally, we investigate how $A_k(N_1, \ldots, N_k)$ behaves when the sizes of $N_1, \ldots, N_k$ start varying and provide a solution to this problem when $k = 3$.

Andrew Ledoan

Zeros of partial sums of the Riemann zeta-function

Abstract: The Riemann zeta-function is that function of the complex variable $s = \sigma + it$, defined in the half-plane $\sigma > 1$ by the absolutely convergent series $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$, which can be continued analytically to a meromorphic function in the complex plane with solely a simple pole situated at the point $s = 1$, with residue 1. The zeta-function is well-approximated unconditionally by arbitrarily short truncations of its Dirichlet series in the region $\sigma > 1$, $|s - 1| > 1/10$ and, on the truth of the Lindelöf Hypothesis, in the region $1/2 < \sigma \leq 1$. Although a great deal is known and conjectured about the distribution of zeros of the zeta-function, little is known about the zeros of its partial sums, $\sum_{n \leq X} n^{-s}$. In this talk I will present some results from my joint work with S. M. Gonek on the distribution of the zeros of partial sums of the zeta-function, in which we estimate the number of zeros up to height T, the number of zeros to the right of a given vertical line, and other aspects of their horizontal distribution.

Steven J. Miller

From Random Matrix Theory to Number Theory

Abstract: Random matrix theory has enjoyed remarkable success in predicting the behavior of zeros of L-functions. We briefly review classical random matrix theory and discuss the three key inputs needed in both subjects: (1) finding the correct scale to study; (2) developing an explicit formula to related what we want to study to what we know; (3) applying an averaging formula to analyze the relevant quantities. We'll analyze the 1-level density for families of Dirichlet L-functions or cuspidal newforms, emphasizing the methods used to study low lying zeros in families of L-functions. If time permits we'll discuss open problems.

Ram Murty

Artin L-functions

Lecture 1: Introduction to Artin L-functions

Lecture 2: The Chebotarev density theorem

Lecture 3: Special values of Artin L-series

Lecture 4: L-series and transcendental numbers

Abstract: After some basic algebraic preliminaries, we will define Artin L-series and derive their elementary properties. Then we discuss what is known and unknown about these L-series. In lecture 2, we will derive the Chebotarev density theorem and give some applications. In lecture 3, we discuss special values, first at $s = 1$, and then at other integral arguments. In the last lecture, we will give some new results concerning the transcendental nature of certain special values of Artin L-series.

Takashi Nakamura

The joint universality and the generalized strong-recurrence for L-functions

Abstract: In this talk, we show the joint universality for a set of Dirichlet L-functions $L(s+idd_l\tau,\chi)_{l=1}^{m}$, where $1 = d_1, d_2, \dots, d_m$ are algebraic real numbers and linearly independent over $\mathbb{Q}$ and $d \in \mathbb{R} \setminus \{0\}$.

From this property, we obtain that $L(s+idd_l\tau,\chi)_{l=j,k}$, where d_j and d_k are two of the above, has a kind of generalized strong recurrence. Roughly speaking, this means that $L(s+idd_j\tau,\chi)$ is uniformly approximated by $L(s+idd_k\tau,\chi)$.

In addition, as a kind of generalization of above theorems, we show the joint universality and the generalized strong recurrence for a set of Dirichlet L-functions $L(s+i\delta_l\tau, chi)_{l=1}2$, where $\delta_1 = 1$, for almost all δ_2.

Moreover, we consider the relation between the generalized strong recurrence and zeros of Steuding and Selberg class L-functions.

Ameya Pitale

Special values of L-functions

Abstract: Special values of L-functions have fascinating arithmetic significance. For example the values of the Riemann zeta function at positive even integers are related to Bernouilli numbers. In particular, up to a power of π, these values are rational. I will try to illustrate some examples of similar special value results related to L-functions of modular forms and their importance. All these special value results fit into a much larger framework - that of Deligne's conjectures. I will talk about my current research (with Ralf Schmidt) on L-functions of Siegel modular forms twisted by $GL(2)$ modular forms and their special values.

Jeremy Rouse

Bounds for the coefficients of powers of the Δ-function

Abstract: Let

$$\Delta^k(z) = q^k \prod_{n=1}^{\infty}(1-q^n)^{24k} = \sum_{n=k}^{\infty} \tau_k(n) q^n.$$

Work of Deligne implies that $|\tau_k(n)| \leq C_k d(n) n^{(12k-1)/2}$ for all n and for some constant C_k. We will compute bounds on C_k as a function of k. An important part of the argument is the non-existence of Siegel zeroes of the symmetric square L-function.

Shenghao Sun

Zeta functions of Artin stacks over finite fields

Abstract: We give the definition of the zeta function of an Artin stack over a finite field, and compute explicitly for the example BG_m, the classifying stack of the multiplicative group. Then we give some general results for these functions: meromorphic continuation and an upper bound for the weights of cohomology groups.

Andreas Weingartner

Zeros of Dirichlet series with periodic coefficients

Abstract: Let $a = (a_n)_{n\geq 1}$ be a periodic sequence, $F_a(s)$ the meromorphic continuation of $\sum_{n\geq 1} a_n/n^s$, and $N_a(\sigma_1, \sigma_2, T)$ the number of zeros of $F_a(s)$, counted with their multiplicities, in the rectangle $\sigma_1 < \Re s < \sigma_2$, $|\Im s| \leq T$. We extend previous results of Laurinčikas, Kaczorowski, Kulas, and Steuding, by showing that if $F_a(s)$ is not of the form $P(s)L_\chi(s)$, where $P(s)$ is a Dirichlet polynomial and $L_\chi(s)$ a Dirichlet L-function, then there exists an $\eta = \eta(a) > 0$ such that for all $1/2 < \sigma_1 < \sigma_2 < 1 + \eta$, we have $c_1 T \leq N_a(\sigma_1, \sigma_2, T) \leq c_2 T$ for sufficiently large T, and suitable positive constants c_1 and c_2 depending on a, σ_1, and σ_2.

Mohammad Zaki

Dirichlet series for weighted convolutions of von Mangoldt function

Abstract: We study the meromorphic continuation of the Dirichlet series

$$F_r(s) = \sum_{\substack{m_1,\dots,m_r \geq 1 \\ m_1 \cdots m_r \equiv b \pmod q}} H(\alpha \log(m_1 + \cdots + m_r)) \frac{\Lambda(m_1) \cdots \Lambda(m_r)}{(m_1 + \cdots + m_r)^s},$$

where Λ is the classical Von Mangoldt function, H is a smooth periodic function, b is any integer, and q and r are positive integers. Assuming GRH, we prove that tis series is analytic in the half plane $\Re(s) > r - \frac{1}{2}$, except for simple poles $s = r + 2\pi i\alpha n$, $n \in \mathbb{Z}$. We also give the resideues at these poles. The most important point of this study is that we show these functions have natural boundaries. For example, we show that if we assume GRH, then the line $\Re(s) = \frac{3}{2}$ is the natural boundary of $F_2(s)$ under some conditions on H and α.