MATH 213 FINAL EXAM STUDY GUIDE

General Information: The Final Exam will in the Testing Center during finals week, Monday Dec. 13 through Friday Dec. 17. The final exam will be cumulative, with a slight emphasis on material you have not been tested on yet (Chapters 5 and 7). Past final exams can be found on Learning Suite. Below is an outline of important points from the later material. Please see the study guides for Exams 1 and 2 for summaries of previous sections.

DISCLAIMER 1: This study guide is NOT intended to be an exhaustive list of everything that could possibly show up on the exam. It is a guide to help you focus your study on the most important things.

DISCLAIMER 2: The practice exam is provided as a source of some good practice problems, and so that you can familiarize yourself with the format of the exam. You should not assume that your exam will have identical or extremely similar problems to what are on the practice exam. In general, simply working through the practice exam is NOT sufficient preparation for your exam.

General Suggestions and Guidelines

- Go into the test calm and confident. You have worked hard and you know this material; this is just your chance to show off your knowledge.
- In general, answers should be simplified as far as they can be. However, I would suggest that once you get to a good answer, go on to the next problem, and then come back if you have time/energy and work on simplifying your answer.
- Do what you feel confident doing first. If you see a problem you don't know how to approach, skip it, and come back later.
- On free response questions, you should show all your work. Show steps in a logical, easy to follow order. Ask yourself, could an average member of the class look at my work and follow what I am doing? (But don't actually show it to an average member of the class while you are taking the test!)
- Use scratch paper. Some problems will not be obvious, and you may have to try a few approaches before you find the right one. You might need a lot of scratch paper!
- Be sure to read the questions carefully and understand what they are asking. On all of your free response answers, ask yourself, have I answered the question that was asked?
- As much as you can, check your work, and think about whether your answer is reasonable. Especially with something like row reduction, it is very easy to make mistakes.

5.1

- Know the meaning of **orthogonal**, **orthogonal set**, **orthogonal basis**, **orthonormal set**, and **orthonormal basis**.
- Know how to compute dot products and understand dot products in terms of transpose (i.e. $u \cdot v = u^T v$).
- Understand that orthogonal sets are linearly independent.
- Know the formula for finding the coefficients of a linear combination when working with an orthogonal basis.
- Given an orthogonal set (or basis), know how to turn it into an orthonormal set (or basis) by normalizing each vector.
- Know the definition of an **orthogonal matrix**, and how to express this with matrix transpose ($Q^T Q = I$).
- Understand the facts about how multiplication by an orthogonal matrix preserves length and inner product and why these are true.
- Be familiar with properties of orthogonal matrices, such as their determinant, what we know about their rows and columns, and the inverse of an orthogonal matrix.

5.2

- Know the definition of **orthogonal complement**.
- Understand that the orthogonal complement is a subspace, and understand basic properties of orthogonal complements.
- Understand the fact $row(A)^{\perp} = null(A)$ and $col(A)^{\perp} = null(A^T)$.
- Given a basis for a subspace, know how to find a basis for the orthogonal complement.
- Know the formulas for **projection** onto a subspace, and for the **component orthogonal** to a subspace for a vector

$$proj_W(\mathbf{y}) = \left(\frac{\mathbf{y} \cdot \mathbf{u_1}}{\mathbf{u_1} \cdot \mathbf{u_1}}\right)\mathbf{u_1} + \cdots + \left(\frac{\mathbf{y} \cdot \mathbf{u_k}}{\mathbf{u_k} \cdot \mathbf{u_k}}\right)\mathbf{u_k}$$

$$perp_W(\mathbf{y}) = \mathbf{y} - proj_W(\mathbf{y})$$

  (CAUTION: these formulas only work if you have an *orthogonal* basis for your subspace).
- Understand orthogonal decomposition (i.e. that given a subspace W every vector in $\mathbb{R}^n$ has a unique representative as the sum of a vector in W and a vector in $W^{\perp}$.

5.3

- Know the formulas for the **Gram-Schmidt process** and when to use this process.
- Given a basis for a subspace, know how to find an orthogonal basis and an orthonormal basis for that subspace.
- You do NOT need to know anything about QR-decomposition.

5.4

- Know the definition of **orthogonally diagonalizable** and **symmetric matrix**.
- Understand that the eigenvalues of a symmetric matrix are real numbers.
- Understand that eigenvectors for different eigenvalues of a symmetric matrix are orthogonal.
- Know the **Spectral Theorem** (both forms, including Spectral Decomposition).

7.4

- Know the definition of **singular values** and how to find them.
- Know how to find the singular value decomposition (SVD) of any matrix.
- Know what **left singular vectors** and **right singular** vectors are and the Outer Product form of the SVD.
- Understand how the SVD relates to the rank, row space, column space, and null space of the matrix.