

The Core topics are those which are subsection headings and possible specific objectives follow.

## 1 Core Topics Math 112, Calculus I

### 1.1 Limits of Functions:

The student will be able to

- Explain intuitively and graphically the concept of limit of a function;
- Recognize the correct definition of limit.
- Be able to use the definition of limit to prove simple limit statements.
- Recall and use limit theorems in evaluating limits;
- Explain and use one-sided limits, limits at infinity, and infinite limits;
- Apply limits to the description of asymptotes of functions.
- Find  $\lim_{x \rightarrow a} f(x)$  for functions which are not defined at  $a$ .

### 1.2 Continuity:

The student will be able to

- Recognize the definition of continuity at a point;
- Explain the graphical interpretation of continuity;
- Understand different types of discontinuities and which can be rewritten so as to be continuous.
- Use continuity in evaluating limits of composite functions;
- Apply the Extreme Value and Intermediate Value theorems and know a correct statement of these theorems.

### 1.3 The Derivative:

The student will be able to

- Explain and apply the graphical interpretation of the derivative as slope;
- Explain and apply the dynamic interpretation of the derivative as rate of change;
- Know the definition of a derivative and be able to use it to compute the derivative of a function.
- Use the differentiation formulas to find the derivative of any elementary function (polynomial, rational, root, exponential, logarithmic, trigonometric, inverse trigonometric, hyperbolic, and all combinations and compositions thereof);
- Recognize and use the common notations for the derivative;

- Recall and use the relationship between differentiability and continuity;
- Use implicit differentiation to find the first derivative of an implicitly defined function.
- Explain and use the interpretations of the second derivative;
- Compute derivatives of higher order.
- Be proficient in all the differentiation techniques, including the product rule, and chain rule

### 1.4 Applications of the Derivative:

The student will be able to

- Recall and explain the meaning of Rolle's Theorem and the Mean Value Theorem;
- Use the derivative to describe the monotonicity of a function;
- Use the second derivative to describe the concavity of a function;
- Use first and second derivative tests to classify extrema;
- Use the derivatives to find critical points, inflection points, and local extrema;
- Use derivatives to aid in sketching, by hand, the graph of a function;
- Solve optimization problems;
- Solve related rates problems;
- Use L'Hôpital's Rules to evaluate limits.

### 1.5 The Definite Integral:

The student will be able to

- Explain and apply the graphical interpretation of the definite integral as area;
- Explain and apply the dynamic interpretation of the definite integral as total change such as: (given the velocity or acceleration, how do you find the displacement?);
- Recognize a correct definition of the definite integral such as the one on Page 382 of Stewart, not the one on page 380;
- Recall and use the definition of the definite integral as a limit of Riemann sums, that is, find what a certain limit of Riemann sums is in terms of an integral;
- Recognize an integral which corresponds to a sequence of Riemann sums;
- Recall and use linearity and interval properties of definite integrals. "Interval properties" are properties pertaining to the interval of integration like

$$\int_a^b f(x) dx = - \int_b^a f(x) dx \text{ and}$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx;$$

- Recall and explain the Fundamental Theorem of Calculus;
- Find derivatives of functions defined as definite integrals with variable limits including situations which will require the use of other rules of differentiation in conjunction with the fundamental theorem of calculus;
- Use the Fundamental Theorem to evaluate definite integrals by antidifferentiation;
- Use a simple substitution to find an antiderivative.
- Use theorems about monotone sequences to assert convergence of a sequence;
- Test a series of constants for conditional or absolute convergence and understand the meaning of absolute and conditional convergence;
- Find the sum of a convergent geometric series and apply it to practical problems;
- Compute Taylor polynomials centered at various points using the formula for Taylor series;
- Recall or compute Taylor series for basic functions, including remainder terms;
- Use the remainder term of a Taylor series to estimate the error in the approximation of the function;
- Know the Maclaurin series for the functions  $e^x$ ,  $\ln(1+x)$ ,  $\sin(x)$ ,  $\cos(x)$  and  $\frac{1}{1-x}$ .
- Find the radius of convergence and interval of convergence of a power series;
- Differentiate and integrate functions expressed as a sum of a power series and understand the statements of the theorems used to do this;
- Recall and compute binomial series.

## 2 Core topics for Math 113

### 2.1 Techniques of Integration:

The student will be able to

- Find antiderivatives of a wide variety of functions, including polynomial, rational, irrational, trigonometric, inverse trigonometric, logarithmic, exponential, and hyperbolic functions and their combinations;
- Find these antiderivatives by hand, using the techniques of integration by substitution, integration by parts, integration by partial fractions and trigonometric substitutions.
- Change limits in a definite integral when changing the variables.
- Demonstrate knowledge of the difference between an integral and an improper integral.
- Deal with both types of improper integrals, those on an unbounded interval and those involving an unbounded integrand.
- Resolve questions of convergence for improper integrals using comparison tests, limit comparison tests, and direct application of the definition of what it means for an improper integral to converge.

### 2.2 Applications of Integration:

The student will be able to

- Use the definite integral to model and resolve problems in physics and geometry, including problems involving area between graphs of functions, mass, arc length, volumes and surface area.
- Use the techniques of finding volumes by slicing and by shells.

### 2.3 Sequences and Series:

The student will be able to

- Recall and use a correct definition of limit of a sequence;
- Recall and use the definition of infinite series and know the difference between an infinite series and sequence;

### 2.4 Parametric Equations and Polar Coordinates:

The student will be able to

- Use parametric equations to represent a wide variety of curves;
- Find arc lengths of parametric curves;
- Transform coordinates and curves between rectangular and polar coordinates;
- Find areas enclosed by polar curves;
- Find arc lengths of polar curves;
- Parameterize ellipses and circles.