

Math 214 Hwk 10

Problem 1. Compute

$$\int_0^2 \int_x^2 x \sqrt{1+y^3} \, dy dx.$$

Problem 2. Compute

$$\int_0^1 \int_y^1 \sin(x^2) \, dx dy.$$

Problem 3. Find the area of the surface given by $z = f(x, y)$ over the region R , where

$$f(x, y) = \sqrt{a^2 - x^2 - y^2} \quad \text{and} \quad R = \{(x, y) \mid x^2 + y^2 \leq a^2\}.$$

Problem 4. Find the mass of the sphere of radius R whose density at a given point is proportional to the distance between the point and the z -axis.

Problem 5. Consider a cone of uniform density, radius R and height h .

- Find the volume of the cone.
- Find the center of mass of the cone.
- Find the moment of inertia of the cone rotating about its azimuth.

$$\text{Answer: } I_0 = \frac{3MR^4}{10}.$$

Problem 6. Use the change of variables to evaluate the double integral

$$\iint_R \frac{\sqrt{x+y}}{x} \, dx dy,$$

where

$$x = u \quad y = uv,$$

and R is the triangle with vertices $(0, 0)$, $(4, 0)$, $(4, 4)$.