Math 214 Hwk 5

Problem 1. *Cylindrical coordinates are given by (ρ, θ, z), where (ρ, θ) are the polar coordinates in the x–y plane. We have the following:*

$$x = \rho\cos\theta, \quad y = \rho\sin\theta, \quad \textit{and} \quad z = z.$$

Let $f(x, y, z)$ be a given. Using the chain rule, find the following in cylindrical coordinates:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}.$$

This is called the Laplacian of f.

Problem 2. *Spherical coordinates are given by (r, θ, ϕ), where $r > 0$ is the radius, $0 \leq \theta < 2\pi$ is the polar angle in the $x - y$ plane, and ϕ is the angle to the z axis. We have the following:*

$$x = r\sin\phi\cos\theta, \quad y = r\sin\phi\sin\theta, \quad \textit{and} \quad z = r\cos\phi.$$

Let $f(x, y, z)$ be a given. Using the chain rule, find the Laplacian in spherical coordinates.

Problem 3 (Extra Credit). *Find the Laplacian in parabolic coordinates (u, v, θ). We have the following:*

$$x = uv\cos\theta, \quad y = uv\sin\theta, \quad \textit{and} \quad z = \frac{1}{2}(u^2 - v^2).$$