Math 311 Hwk 4

Show your work. Provide both code and output for programming problems.

Problem 1 (15 points). *Let $V = \mathbb{C}^2$, $S = \{e^{i\theta}, e^{-i\theta}\}$ and $T = \{\cos\theta, \sin\theta\}$.*

(a). *Find the transition matrix P from S into T.*

(b). *Find the inverse of P.*

(c). *Write the vectors in T as linear combinations of elements of S.*

Problem 2 (25 points). *Let $V = \mathbb{R}^2$. As represented in the standard basis $\{\mathbf{e}_1, \mathbf{e}_2\}$, let*

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{v}_1 = \begin{pmatrix} 5 \\ 6 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 7 \\ 8 \end{pmatrix}.$$

(a). *Find the transition matrix U from $\{\mathbf{u}_1, \mathbf{u}_2\}$ to $\{\mathbf{e}_1, \mathbf{e}_2\}$.*

(b). *Use your answer from part (a) to show that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is a basis for $\mathbb{R}^2$.*

(c). *Find the transition matrix V from $\{\mathbf{v}_1, \mathbf{v}_2\}$ to $\{\mathbf{e}_1, \mathbf{e}_2\}$.*

(d). *Use your answer from part (c) to show that $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for $\mathbb{R}^2$.*

(e). *Use parts (a)–(d) to find the transition matrix from $\{\mathbf{u}_1, \mathbf{u}_2\}$ to $\{\mathbf{v}_1, \mathbf{v}_2\}$.*

Problem 3 (20 points). *Let W be the subspace of $C[a, b]$ spanned by $S = \{e^x, xe^x, x^2e^x\}$ and let $L[f(x)] = \frac{df}{dx}$.*

(a). *Find the matrix D representing L on S.*

(b). *Find the inverse of D.*

(c). *Use your answer in part (b) to find the anti-derivative of*

$$f(x) = ae^x + bxe^x + cx^2e^x.$$

(d). *Use your answer in part (c) to find*

$$\int_a^b 3xe^x - 7x^2e^x dx.$$

Problem 4 (15 points). *Let W be the subspace of $C[a, b]$ spanned by the set $S = \{1, e^x, e^{-x}\}$, and let $L : W \longrightarrow W$ be given by $L[f] = \frac{df}{dx}$.*

(a). *Find the matrix representation A of L in the basis S.*

(b). *Find the transition matrix P from $T = \{1, \cosh x, \sinh x\}$ into S.*

(c). *Using similarity, find the representation B of L in the basis T.*

Problem 5 (10 points). *Write a Malab function called* `QR` *that performs QR-decomposition. You can assume that the $m \times n$ input matrix is rank n. Verify your program by decomposing random matrices A into Q and R and then multiplying them back to get A.*

Problem 6 (15 points). *Write a Matlab function called* `ip(p,q,b,a)` *that will compute the inner product for polynomials $p(x)$ and $q(x)$ of arbitrary degree on the interval $[b, a]$ via*

$$\langle f(x), g(x) \rangle = \int_a^b f(x)g(x)dx.$$

Hint: Represent the polynomials $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ as row vectors $[a_n \; a_{n-1} \; \ldots \; a_1 \; a_0]$.