

Math 311 Hwk 5

Show your work. Provide both code and output for programming problems.

Problem 1 (25 points). Consider the basis $\{1, x, x^2, \dots\}$ of $\mathbb{R}[x]$. Find the first 4 terms (by hand) of the orthonormal basis obtained by Gram Schmidt orthogonalization. Use the inner product

$$\langle f, g \rangle = \int_{-1}^1 f(x)g(x) dx.$$

Problem 2 (25 points). Write a Matlab function that will numerically compute the first n orthonormal polynomials from above problem.

Problem 3 (25 points). Orthogonally project the function $f(x) = e^x$ onto $\mathbb{R}^5[x]$. Use the inner product from the first problem. Is your answer different than the 5th-order Taylor expansion of e^x ? Explain why or why not.

Problem 4 (25 points). Write a Matlab script that generates n data points of a noisy circle of radius 10, centered at $(3, 5)$. Then use least squares to “deduce” the radius and center. How close do you get to the known values as the number of data points n increases? Do you recover the original values? Explain why or why not. Your answer may depend on how you generated your data points.